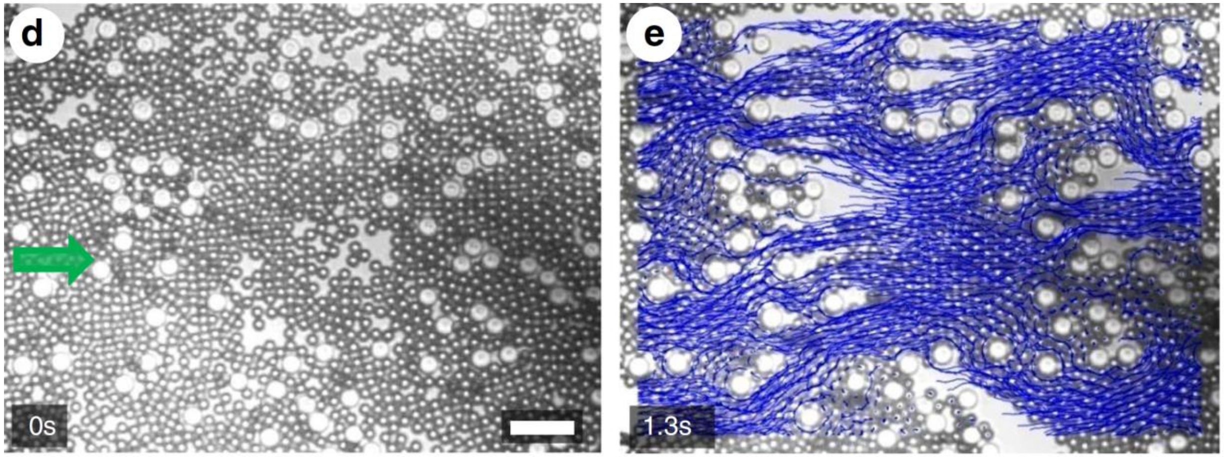
Clogging and jamming of colloidal monolayers driven across a disordered landscape

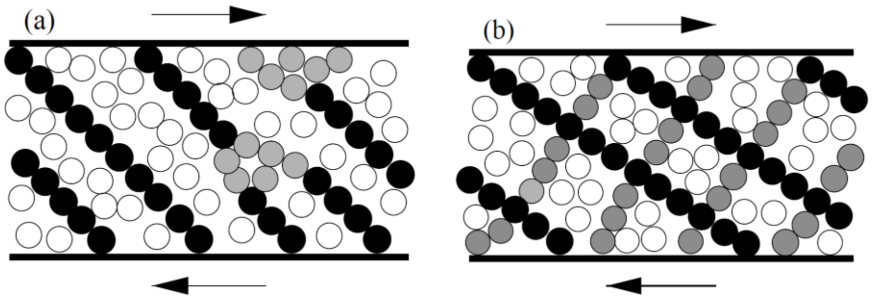


Ralph Stoop, Pietro Tierno

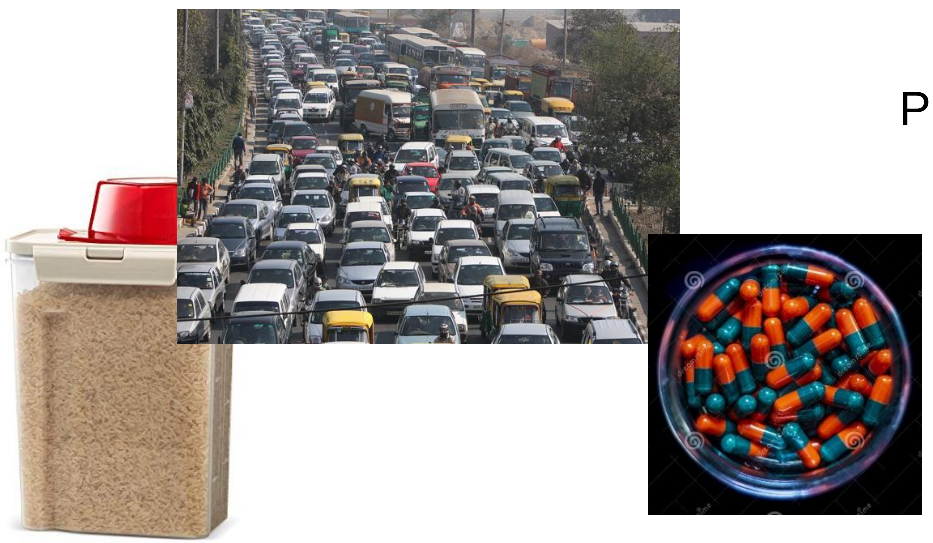
Department of Condensed Matter Physics, University of Barcelona
Institute of Nanoscience and Nanotechnology IN^{2}UB
University of Barcelona Institute of Complex System, UBICS

Jamming

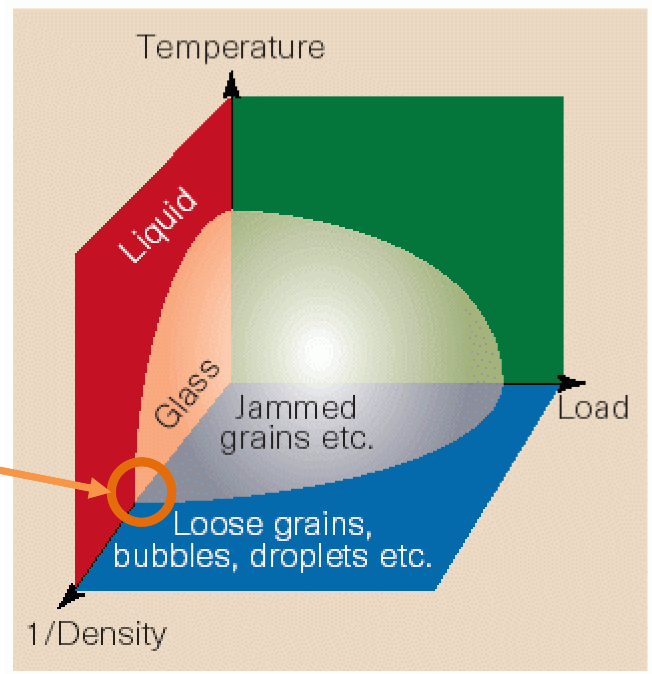
→ Development of a resistance to shear



M. E. Cates, PRL 81, 1841 (1998)



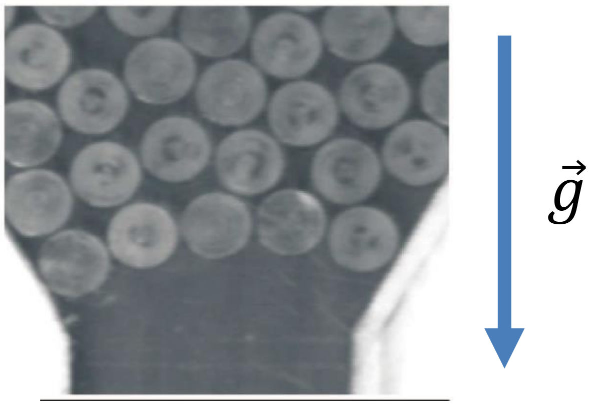
Point J



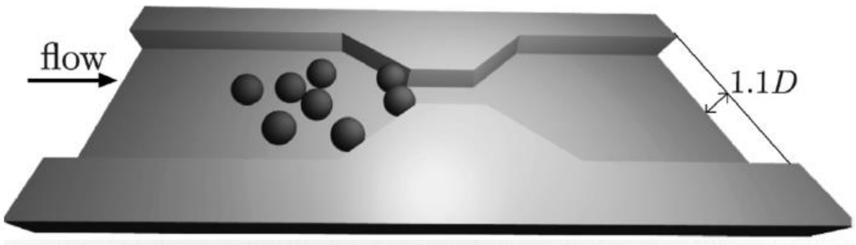
A. J. Liu, S. R. Nagel, Nature 396, 21 (1998)

Clogging

→ Obstruction of a constriction due to particle flow

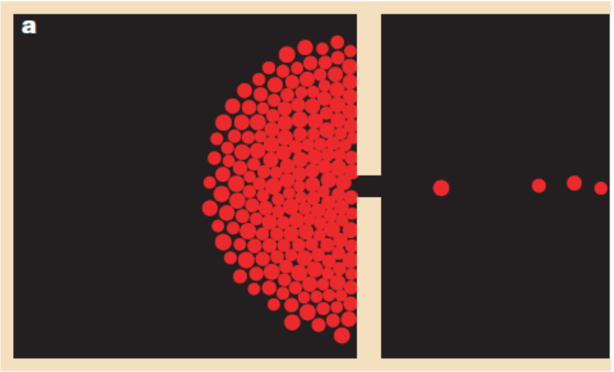


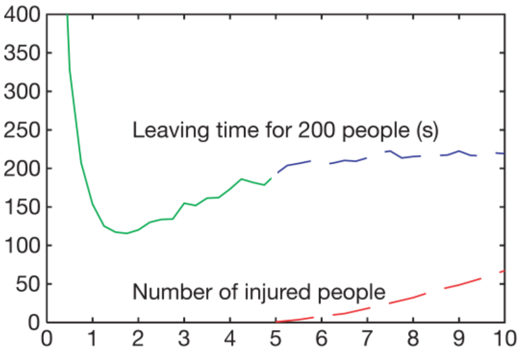
K. To et al. PRL 86, 71 (2001).



A. Marin et al PRE(R), 97 021102 (2018).

"Faster is slow effect" in escape dynamics

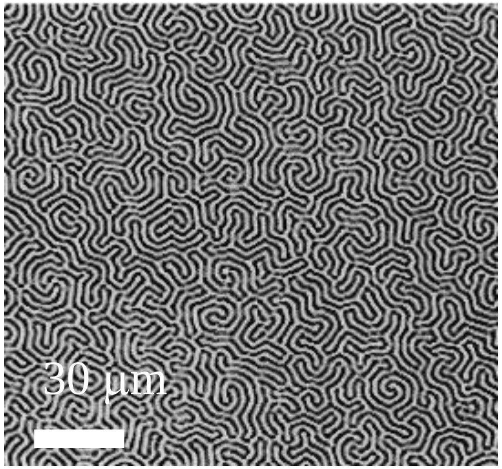




Above v_0=5 m/s, people are injured and become non-moving obstacles for others

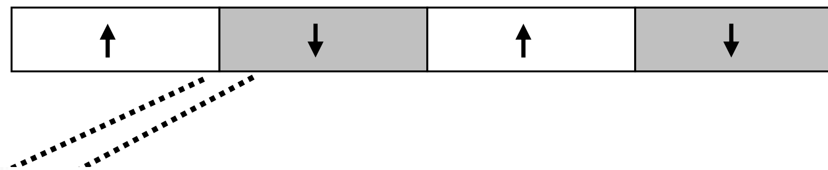
D. Helbing et al, Nature, 407, 487 (2000).

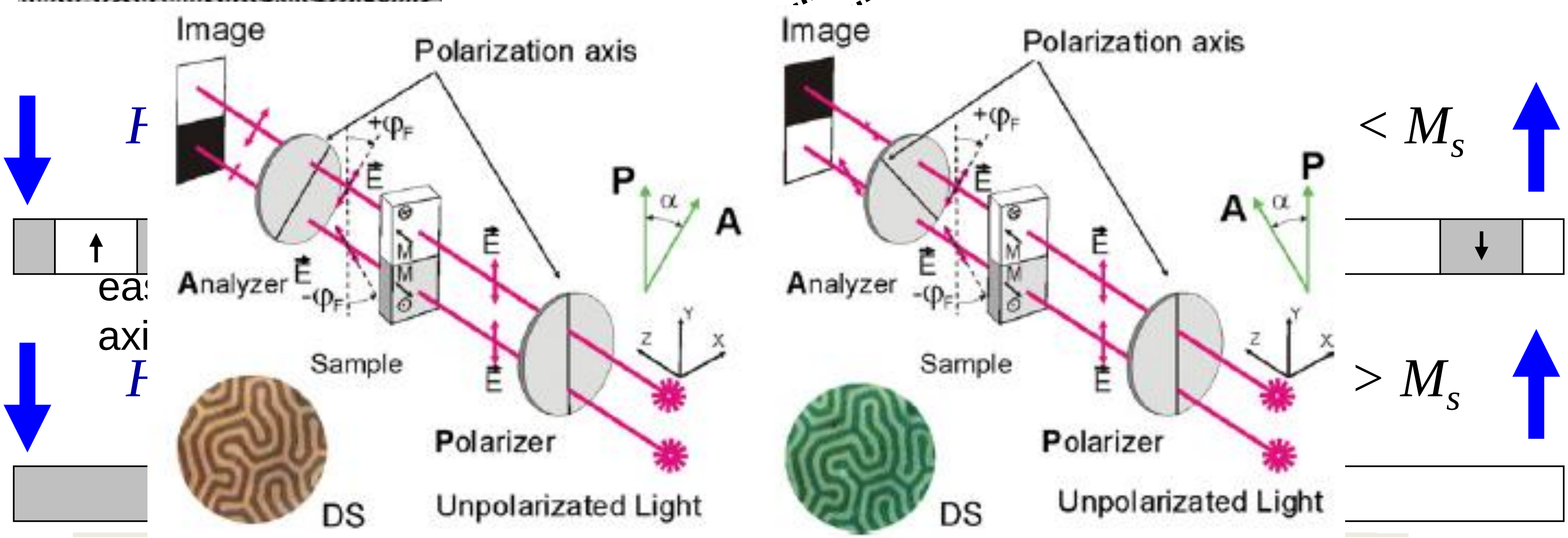
Magnetic periodic potential



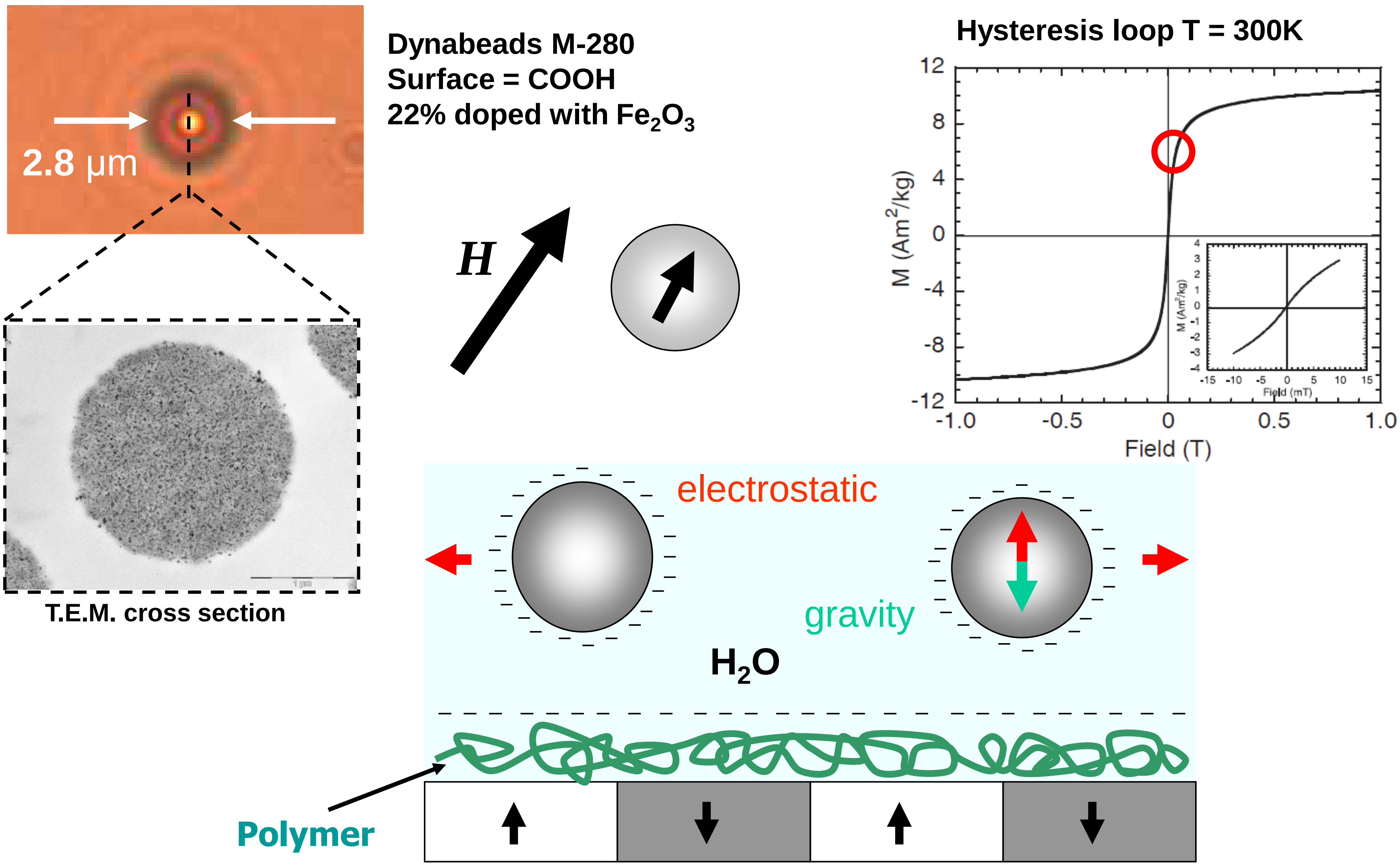
uniaxial ferrite garnet film (FGF)
$Y_{2.5}Bi_{0.5}Fe_{5-q}Ga_qO_{12}$ (q = 0.5-1)
thickness ~ 4 μm
M_s = 1.7·10^4 A/m.
λ = 11.6 μm

$H_{ext} = 0$

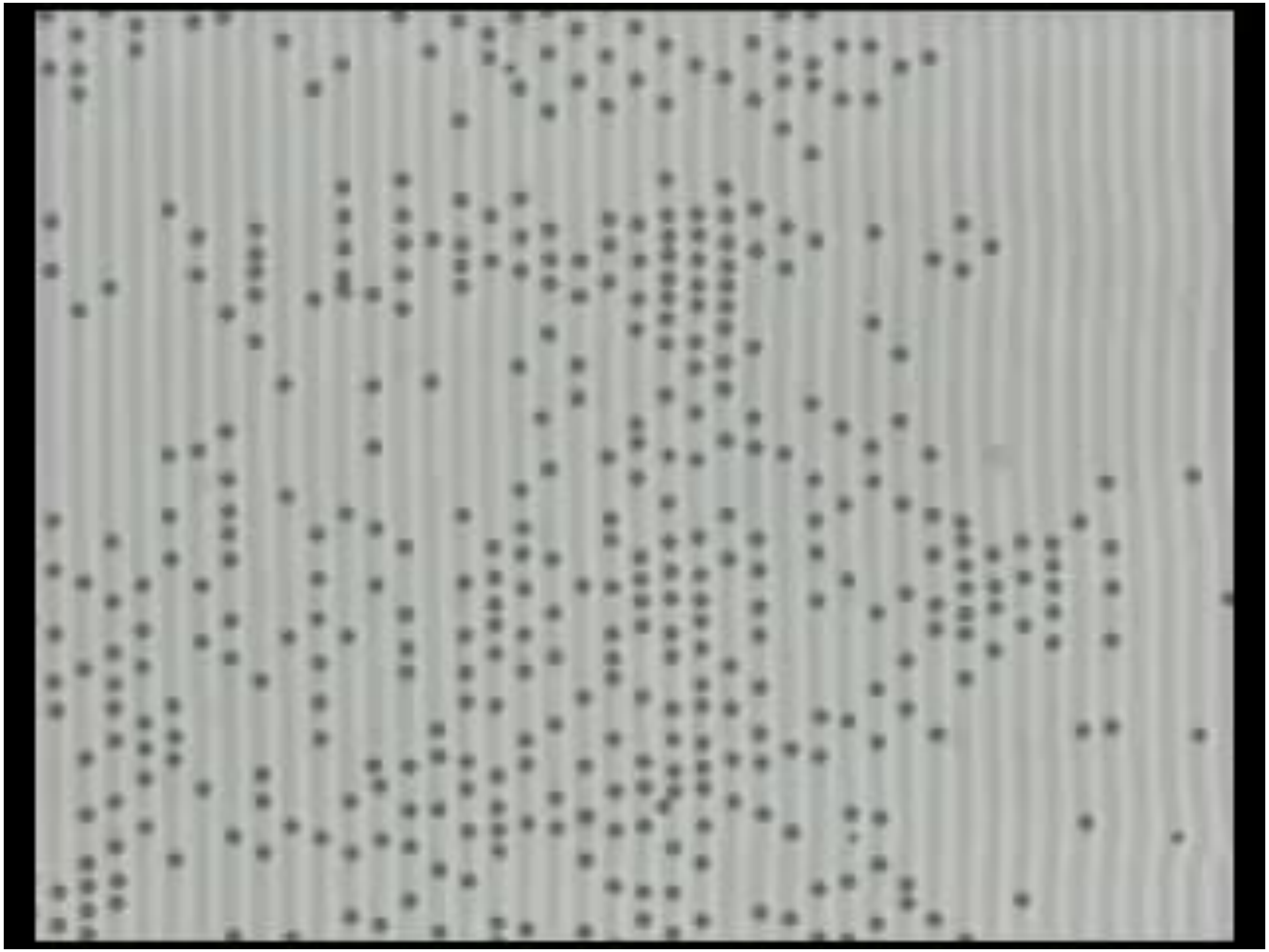




Paramagnetic colloids



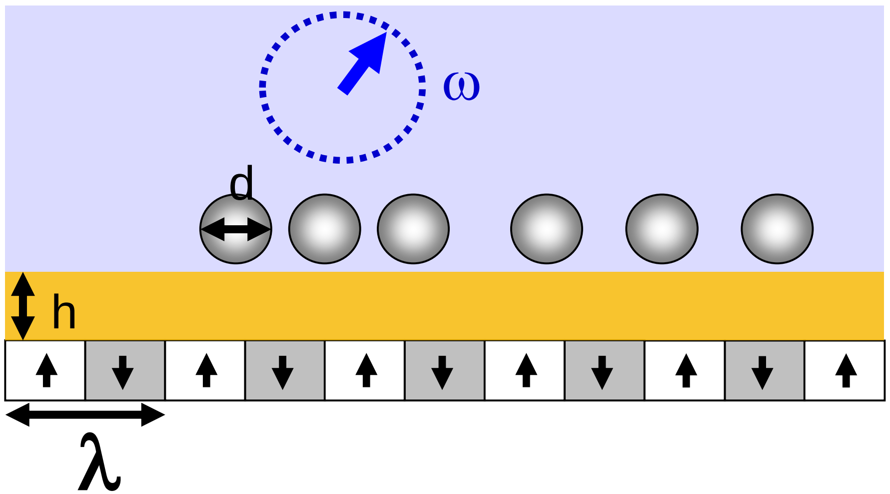
Traveling wave ratchet



P. Tierno et al., *JPCB 111, 13097; JPCB 111, 13479; JPCB 112, 3833; PRE, 75, 041404*

Traveling wave ratchet

$$H(t) \equiv [H_x \cos(\omega t), 0, -H_z \sin(\omega t)]$$

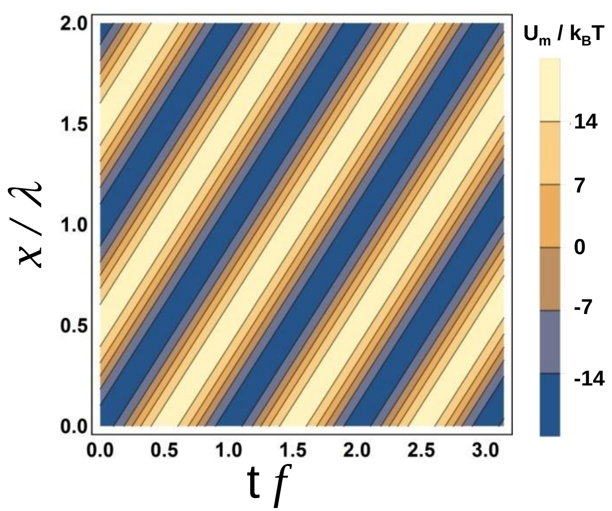


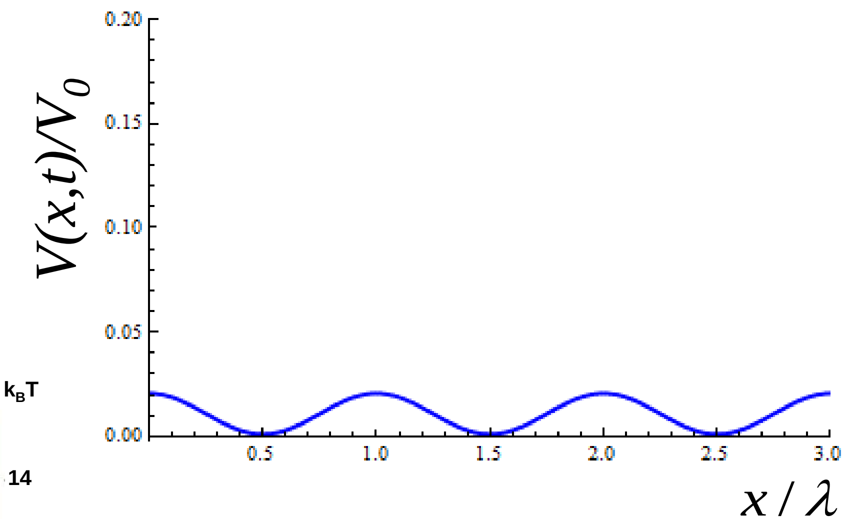
values

$h = d = 1\mu m$

$\lambda = 2.6\mu m$

$V \sim 5k_B T$





Model single particle (with A. V. Straube, FU Berlin)

- Overdamped Langevin eq.:

$$\zeta \dot{x}(t) = -\frac{\partial V(x,t)}{\partial x} + \sqrt{2\zeta k_B T}\,\xi(t)$$

Potential (approx.)

$$\frac{V(x,t)}{V_0} = -\frac{8H_0}{\pi M_s} e^{-2\pi z/\lambda} \cos\left(\frac{2\pi x}{\lambda} - \omega t\right)$$

new variable $\quad y(t) = -x(t) + \Omega t / 2\pi$

Stochastic Adler eq.

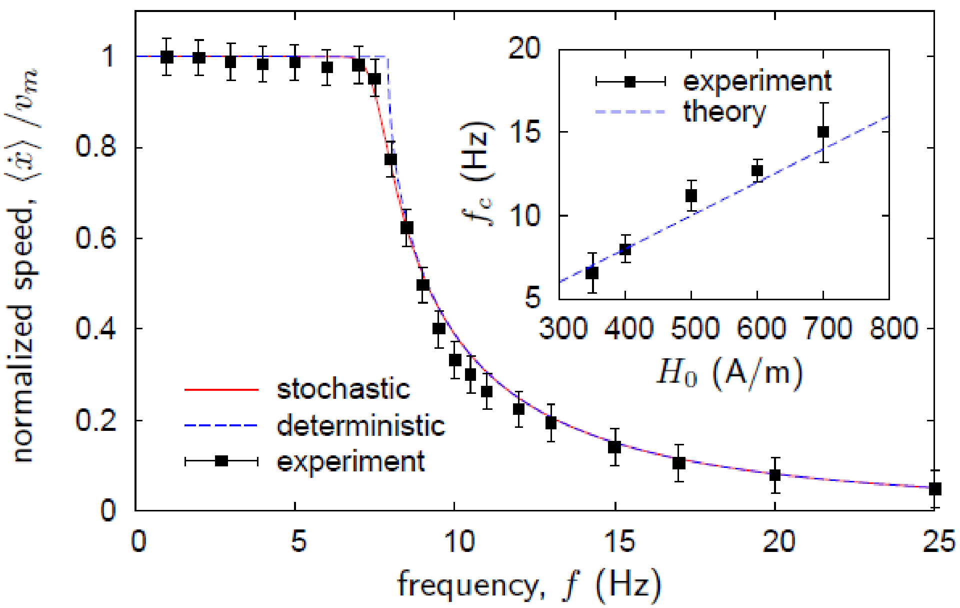
$$\dot{y}(t) = \left(\frac{\Omega - \Omega_c}{2\pi}\right) \sin[2\pi y(t)] + \cancel{\sqrt{2\sigma}\,\xi(t)}$$

deterministic solution

$$\frac{\langle \dot{x} \rangle}{v_m} = \begin{cases} 1 & \Omega < \Omega_c \\ 1 - \sqrt{1 - (\Omega_c / \Omega)^2} & \Omega > \Omega_c \end{cases}$$

$\Omega_c = 16 H_0 e^{-2\pi z}$ critical frequency

$\zeta = 6\pi\eta a$	friction coeff.
$V_0 = (4\pi a^3/3)\chi\mu_s M_s^2$	potential strength
$\sigma = k_B T / V_0$	noise strength
$\Omega = (\omega\zeta\lambda^2)/(2\pi V_0)$	dimensionless freq
$v_m = \lambda\omega / 2\pi$	max. speed



A. V. Straube, P. Tierno *EPL* **103**, 28001 (2013)

Model single particle

- thermal fluctuations: Fokker-Planck eq.

$$\frac{\partial}{\partial t}P(y,t)=2\pi\sigma\frac{\partial}{\partial y}\left[\frac{dV(y)}{dy}+\frac{1}{2\pi}\frac{\partial}{\partial y}\right]P(y,t)$$

stationary solution

$$P_0(y)=\frac{e^{-V(y)}}{e^{-\pi D}\left|I_{iD}(D_c)\right|^2}\int_y^{y+1}e^{V(y')}dy'$$

$$D=\Omega/(4\pi^2\sigma)$$

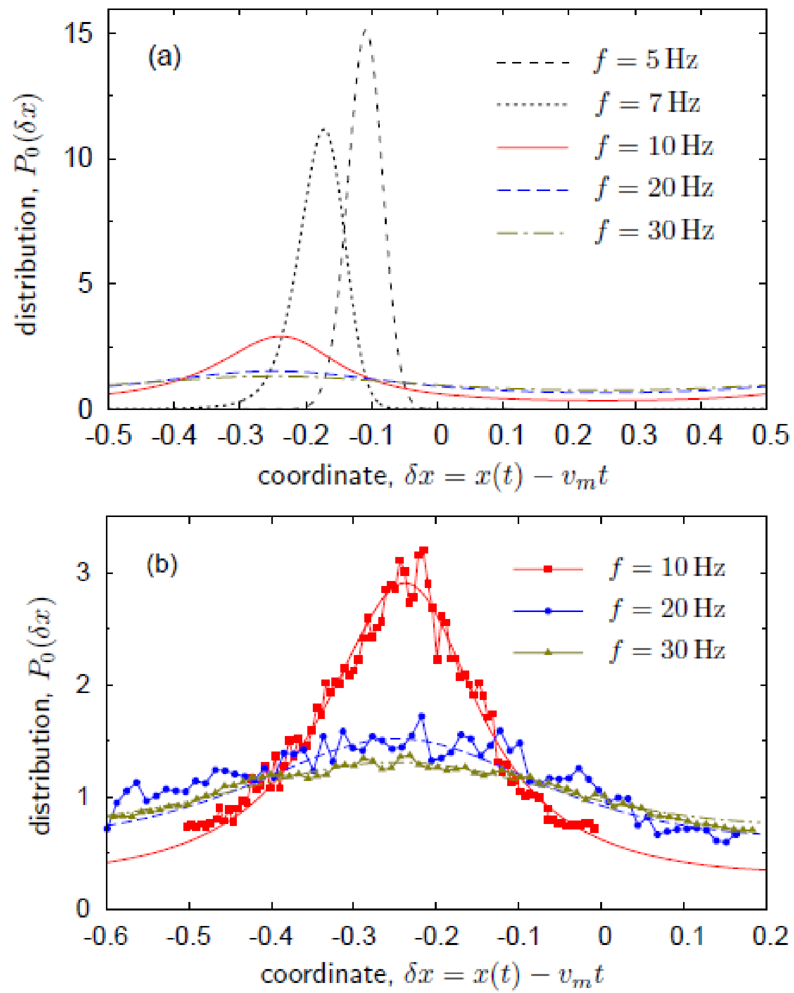
$$D_c=\Omega_c/(4\pi^2\sigma)$$

modified Bessel function
the first kind of an
imaginary order

$$\langle\dot{y}\rangle=\int_0^1\dot{y}P_0(y)dy$$

stochastic solution

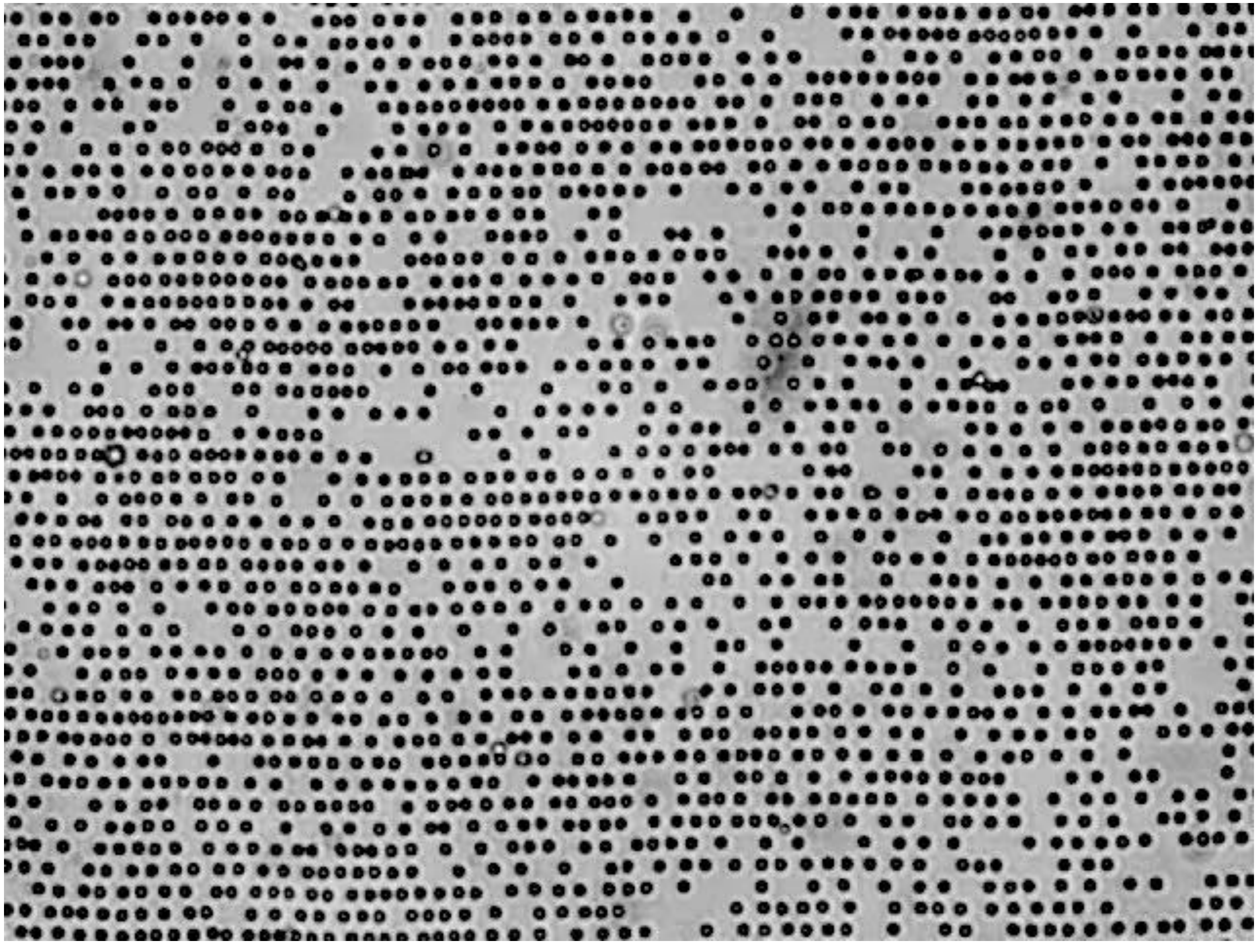
$$\frac{\langle\dot{x}\rangle}{v_m}=1-\frac{\sinh(\pi D)}{\pi D\left|I_{iD}(D_c)\right|^2}$$



A. V. Straube, P. Tierno *EPL* **103**, 28001 (2013)

Depinning and collective dynamics states

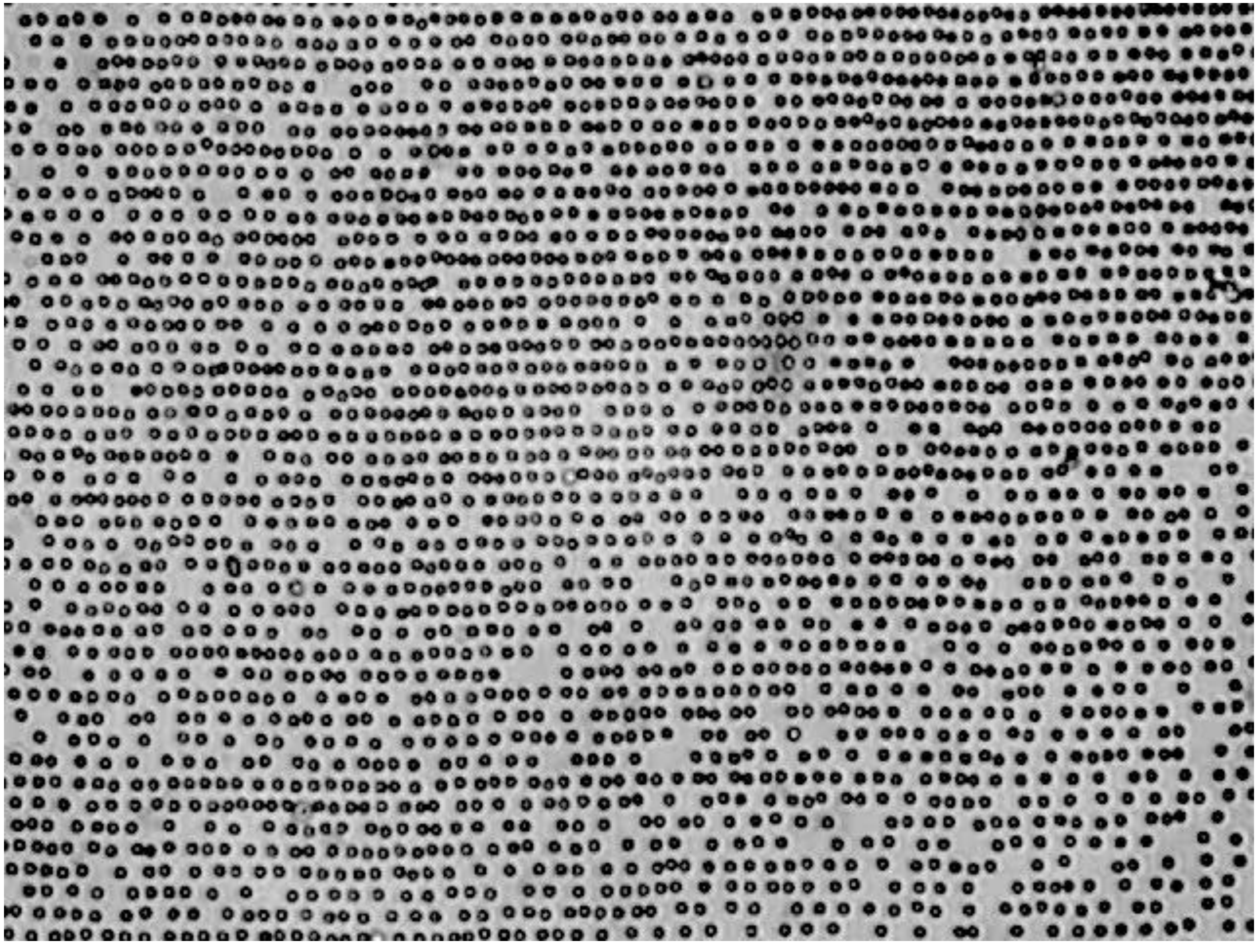
Synchronous: locked smectic flow



P. Tierno *PRL* **109**, 198304 (2012)

Depinning and collective dynamics states

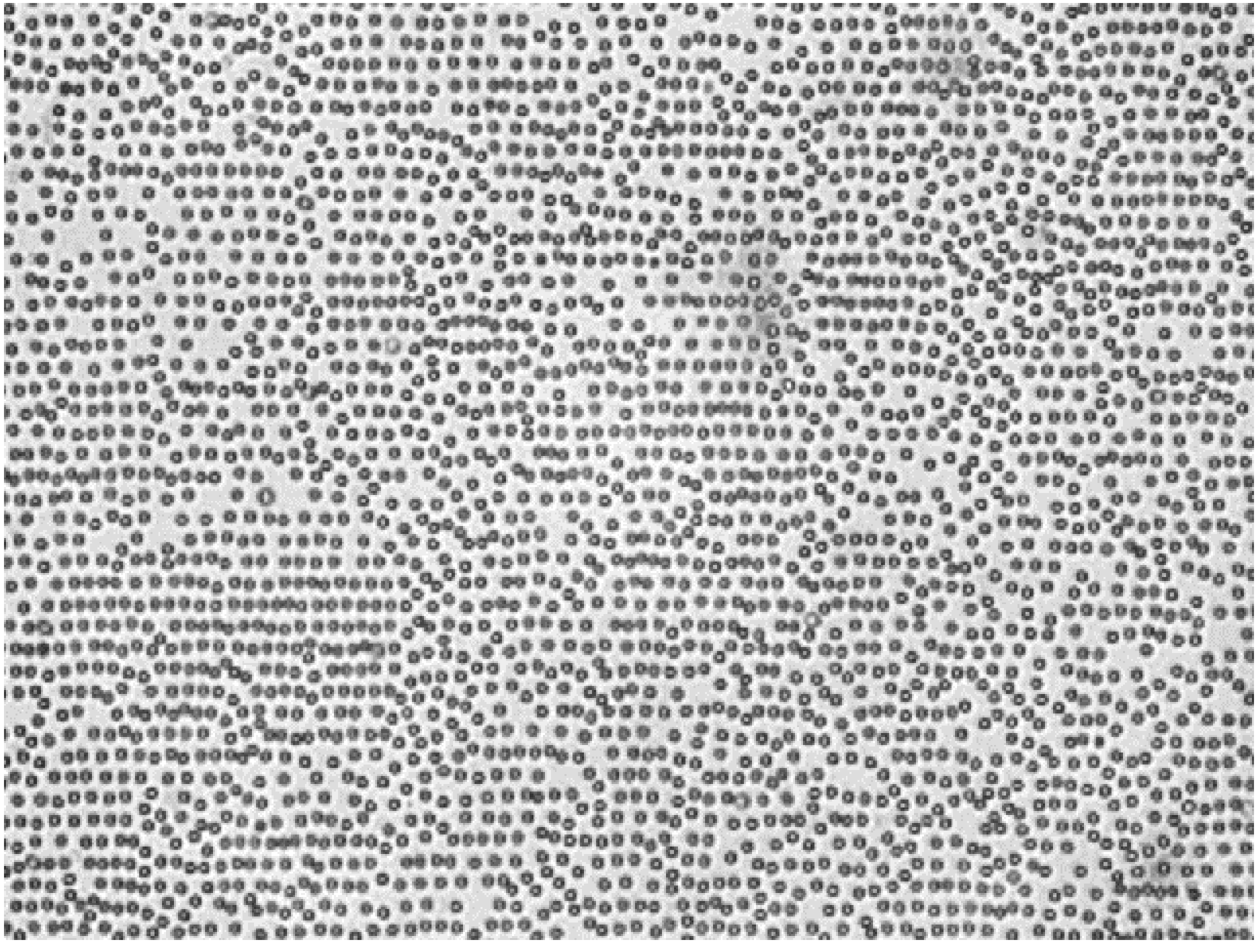
Asynchronous: disordered flow



P. Tierno *PRL* **109**, 198304 (2012)

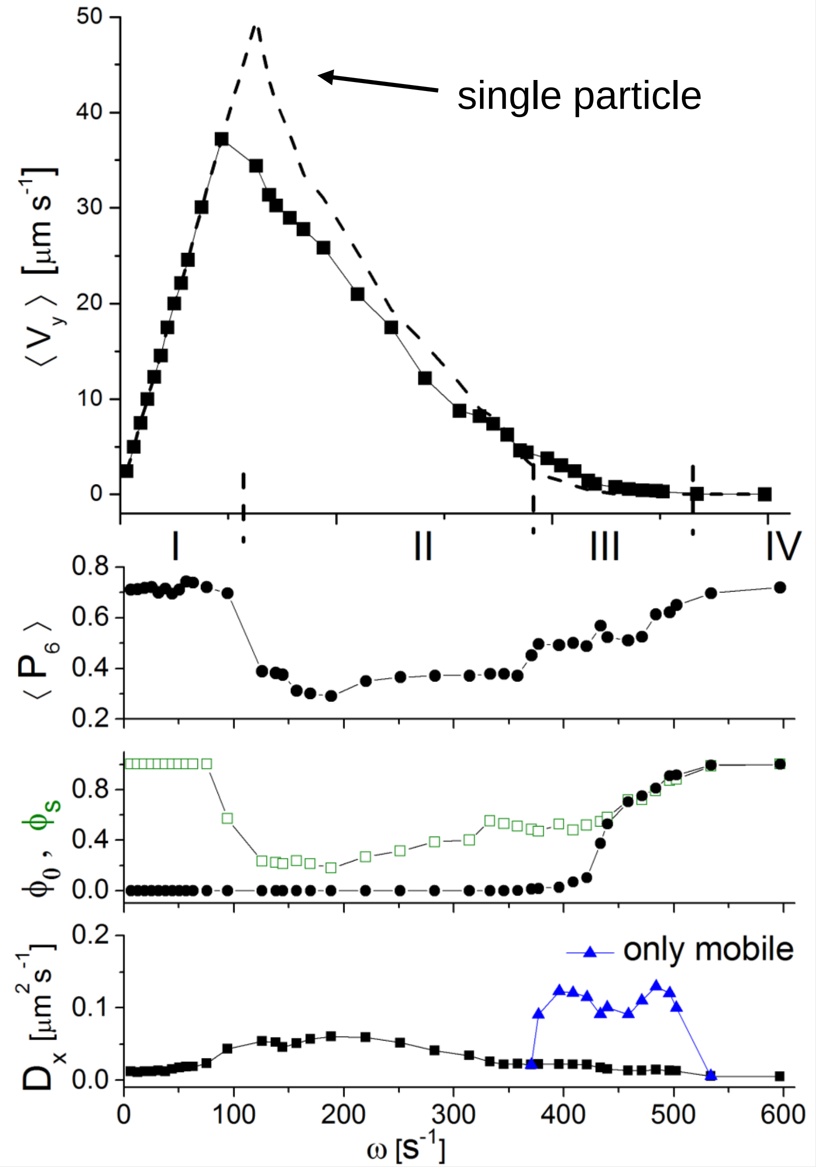
Depinning and collective dynamics states

Asynchronous: two phase-flow



P. Tierno *PRL* **109**, 198304 (2012)

Depinning and dynamics states



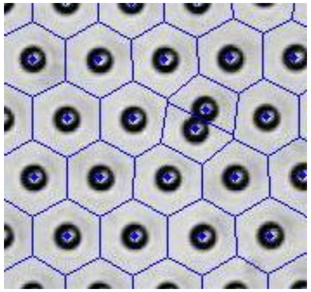
Statistical tools

- average velocity along the driving (*y*-) direction

$$\langle v_y(t) \rangle = \left\langle \frac{1}{N} \sum_i \frac{dy_i}{dt} \right\rangle$$

- fraction of sixfold coordinated particles

$$\langle P_6 \rangle = \left\langle \frac{1}{N} \sum_i \delta(z_i - 6) \right\rangle$$



- fraction of colloids moving with the same velocity (with zero velocity)

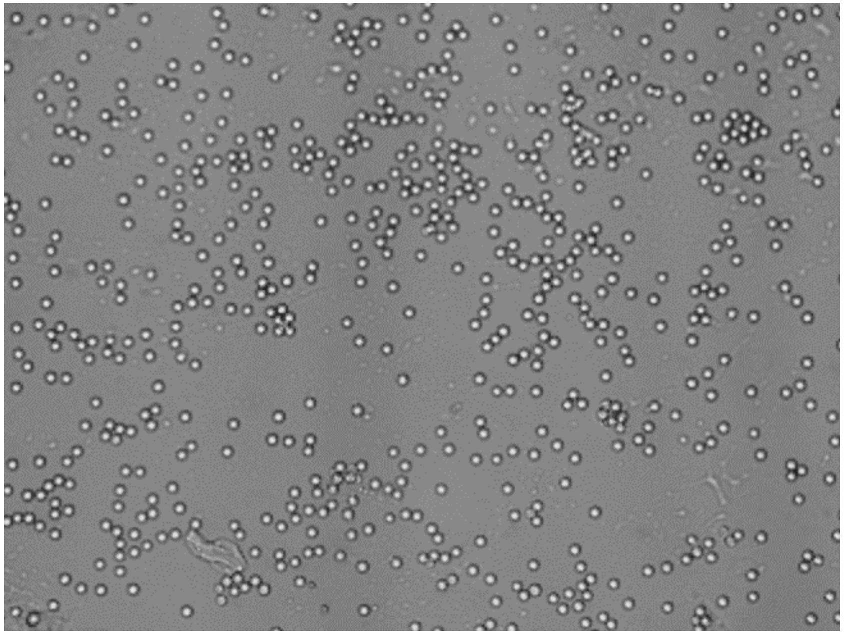
$$\phi_s(\phi_0)$$

- transversal diffusion coefficient

$$D_x = \langle [x_i(t) - x_i(0)]^2 \rangle / 2t$$

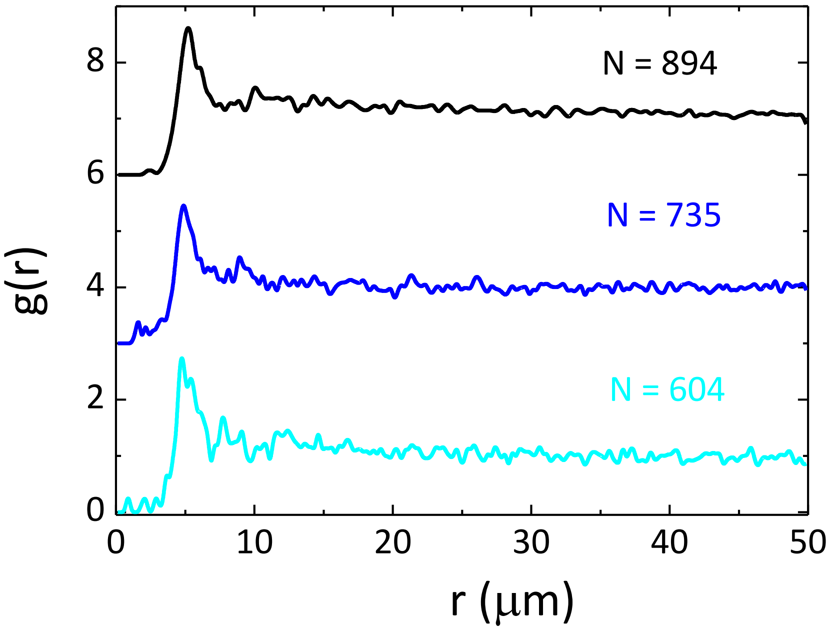
Quenched disorder of obstacles

Large silica sphere (5 μm diameter)



Deposited and attached to the magnetic substrate

Liquid like structure

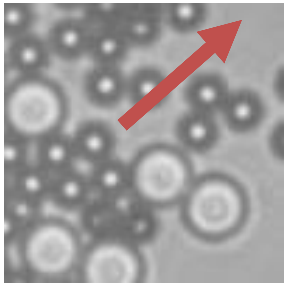


$$\Phi_j = \frac{N_j \pi (d_j)^2}{4A_0}$$ $j = o$ (obstacles), m (magnetic colloids)

$$\Phi_m = \frac{\pi}{2\sqrt{3}}$$ ~0.9 Close packing

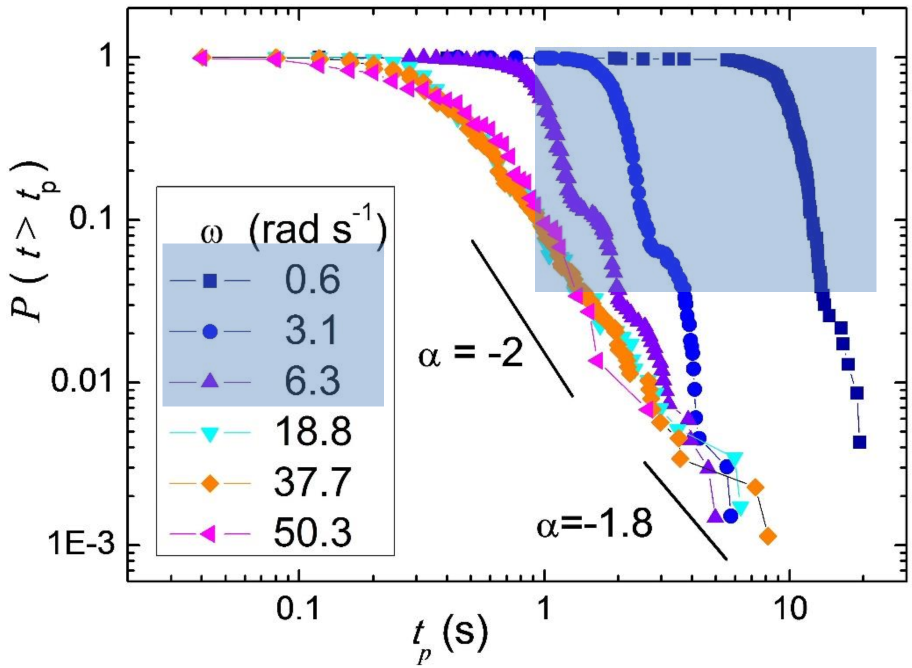
N_j Number of elements j with diameter d_j A_0 = observation area

Transport trough one aperture



Distribution P(t > t_p) of time lapse t > t_p between the particles passing through the aperture

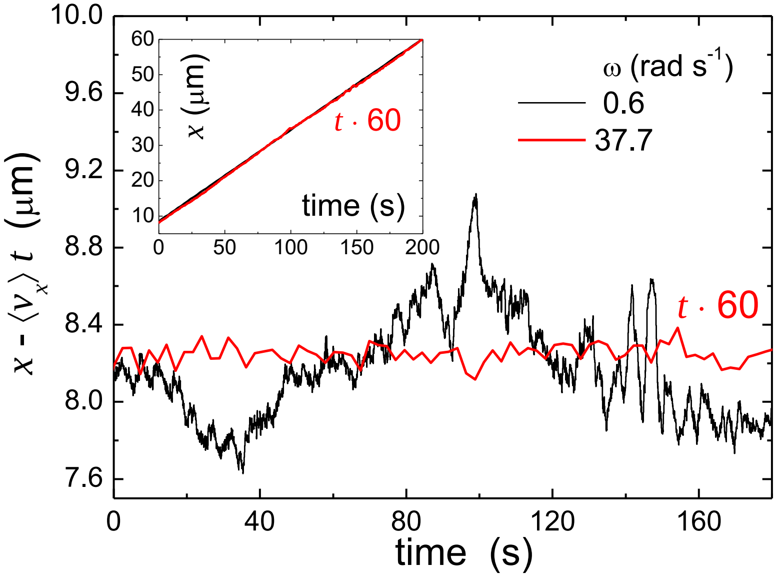
$$t_p = 0.4\mathrm{s}$$



Power law tail at high frequency $P \sim t^{-\alpha}$

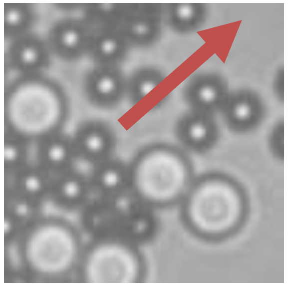
Clog-free system $\alpha > 2$

low frequency ω



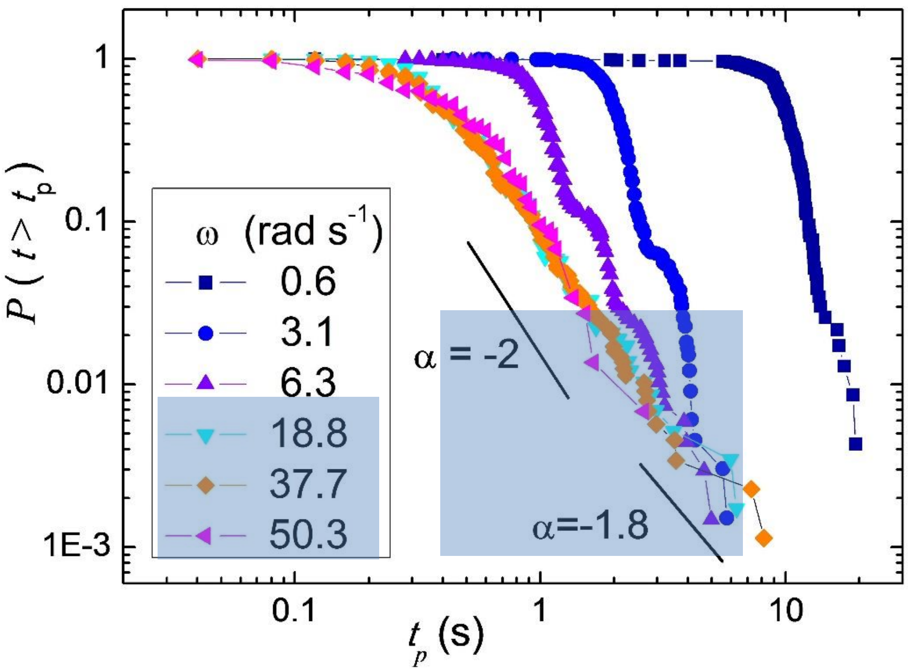
Strong particle vibrations on the garnet film reduce clogging

Transport trough one aperture



Distribution $P(t > t_p)$ of time lapse $t > t_p$ between the particles passing through the aperture

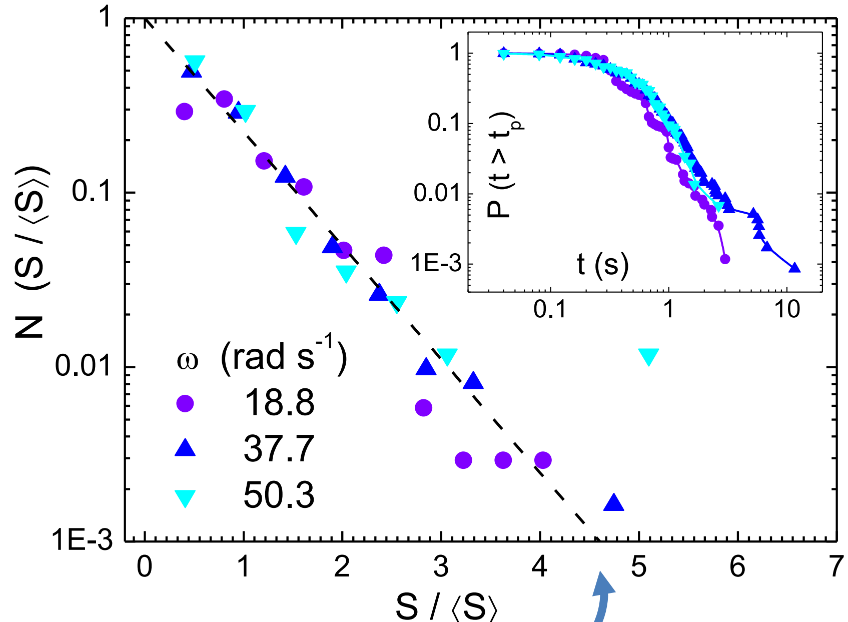
$t_p = 0.4\text{s}$



Power law tail at high frequency $P \sim t^{-\alpha}$

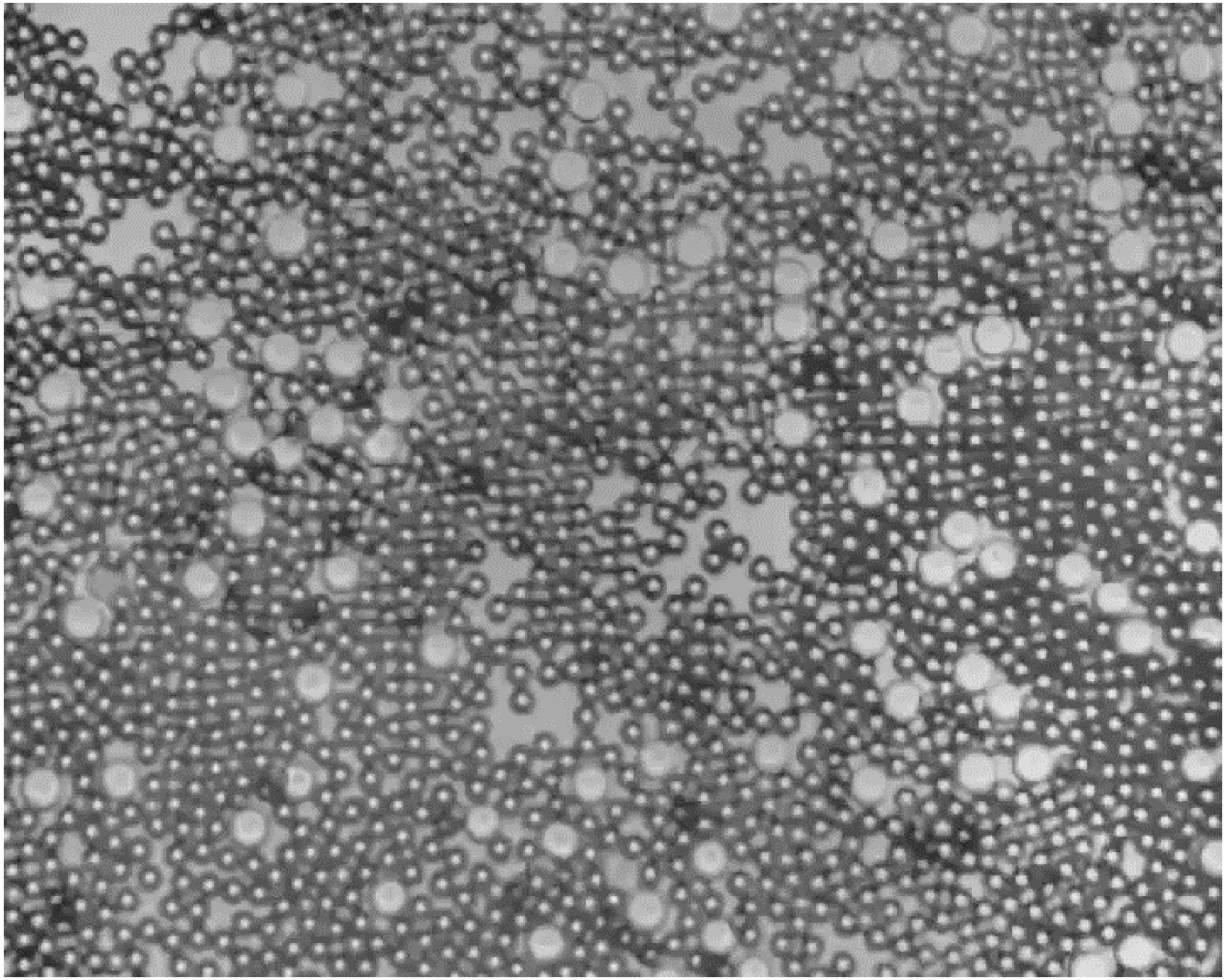
Clogging $\alpha \lesssim 2$

Histograms of burst size



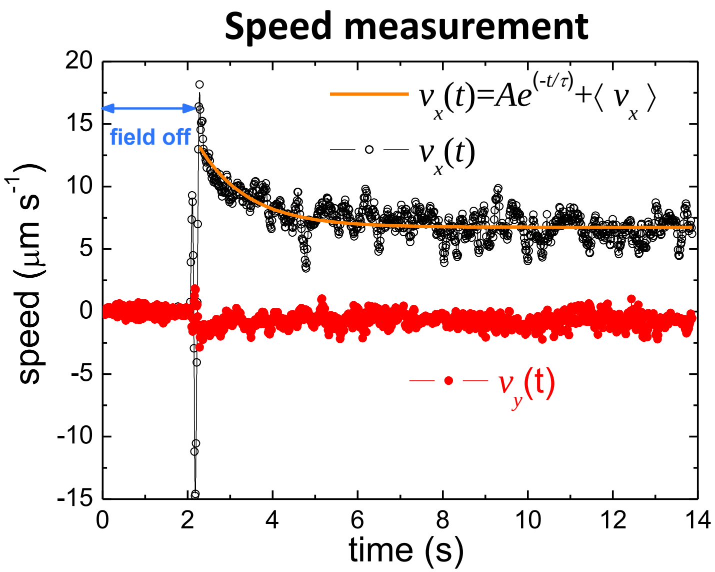
exponential distribution of burst size

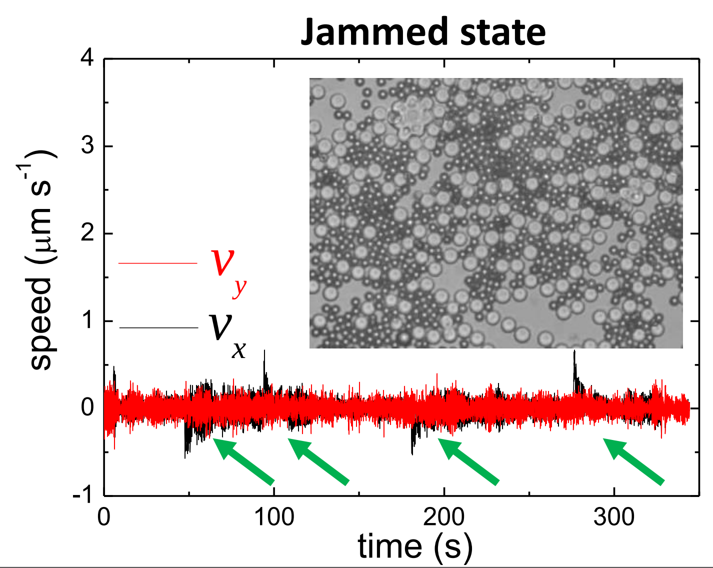
Collective transport

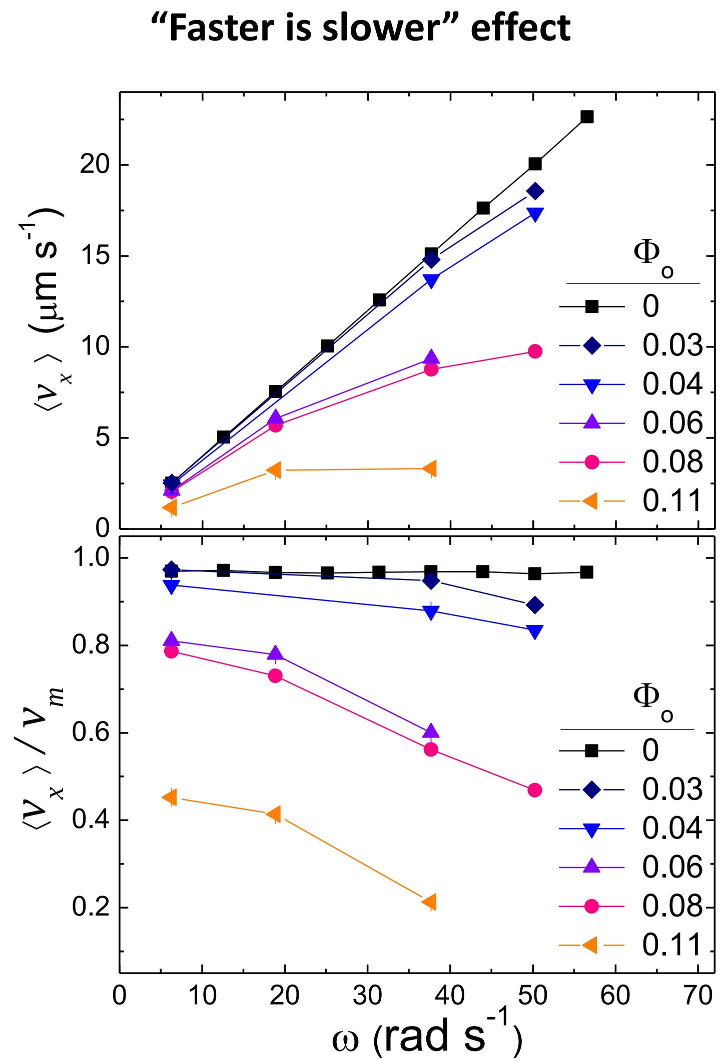


$\langle v_x \rangle \in [2, 23.5]\ \mu\mathrm{m\,s^{-1}} \quad Pe \in [20, 235]$

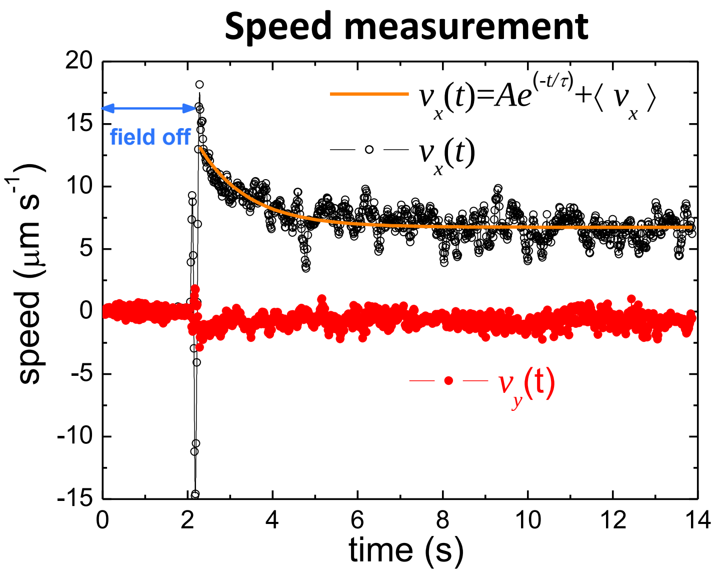
Collective transport

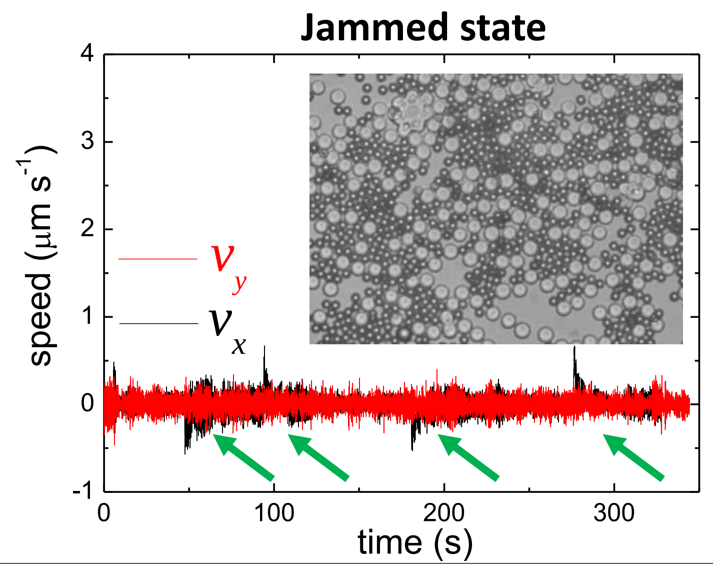


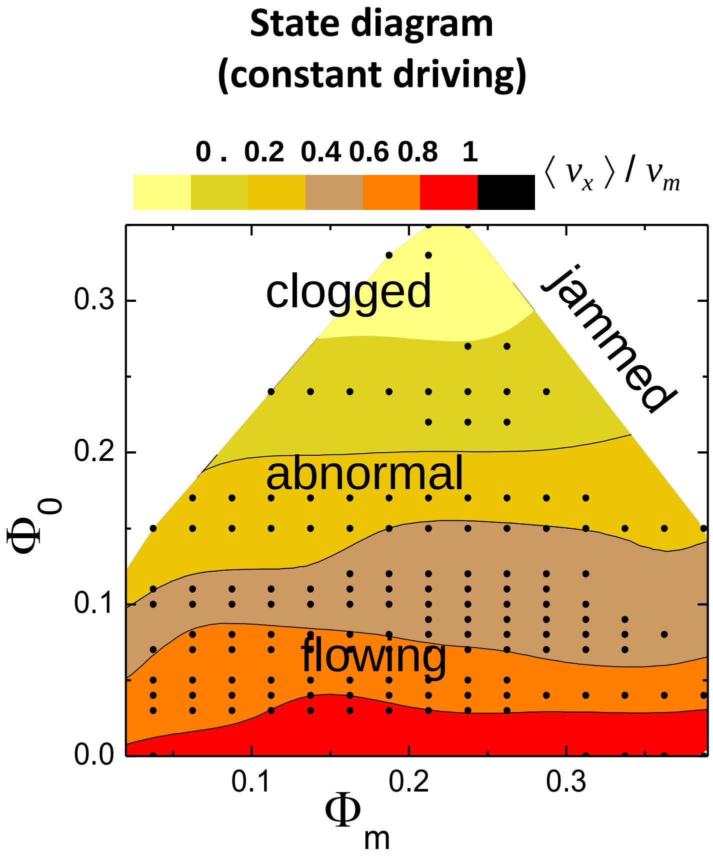




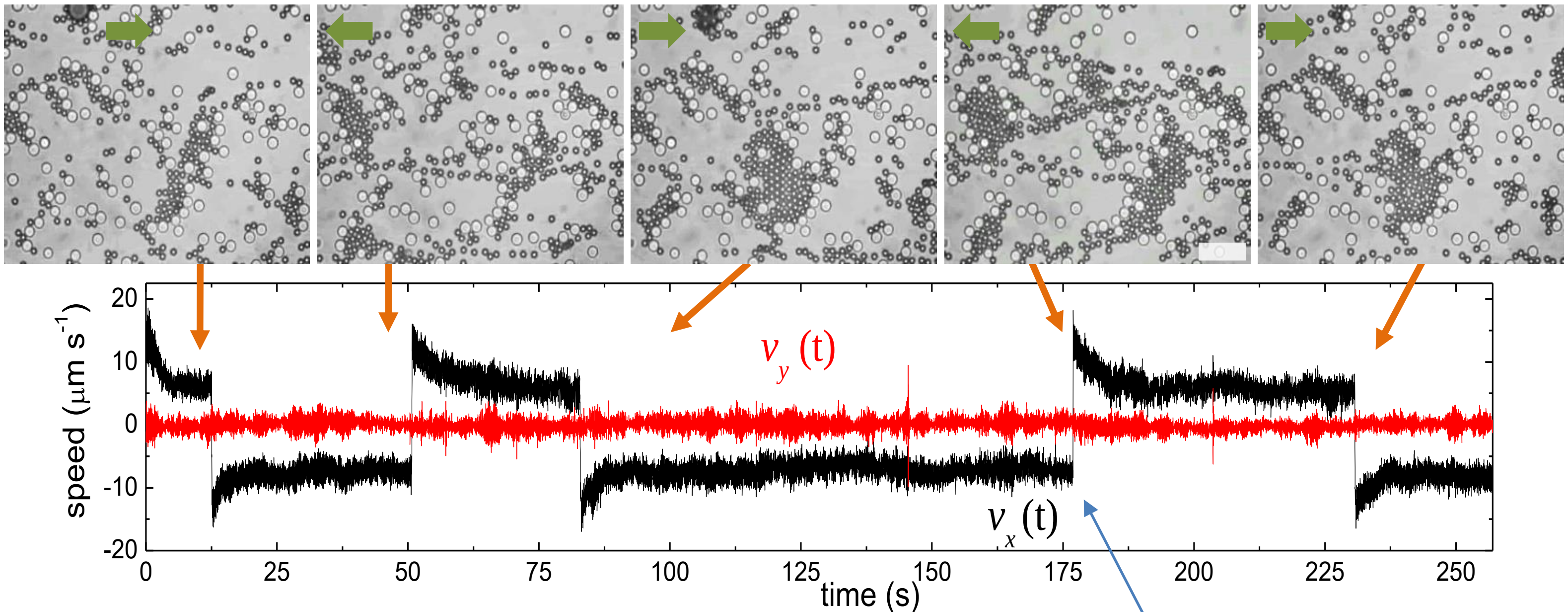
Collective transport



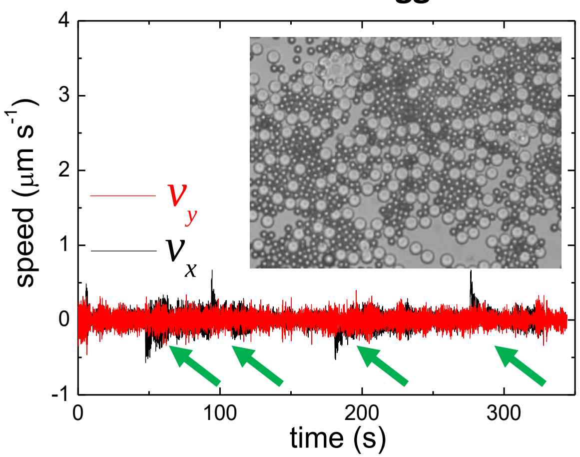




Jamming vs Clogging



Jammed state ≠ Clogged state



$$H_x \leftrightarrow -H_x$$

Field inversion

R. Stoop, P. Tierno *Comm. Phys.* **1**, 68 (2018)

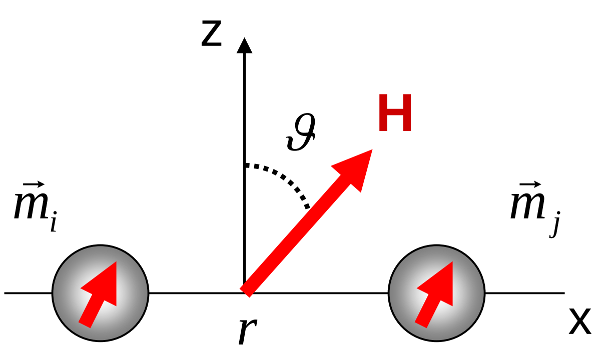
Tuning interactions

Magnetic dipolar interactions $\vec{m}_i, \vec{m}_j$

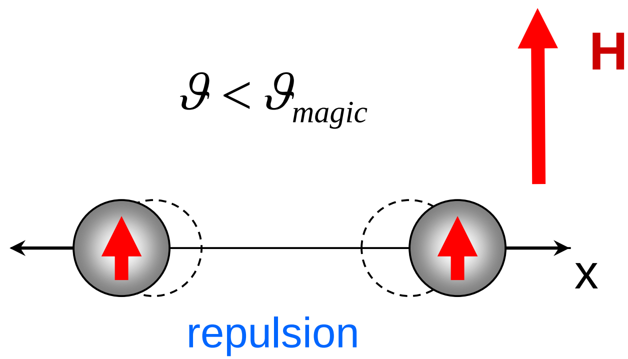
$$U_{ij}^d = \frac{\mu_0}{4\pi r^3}\left[(\vec{m}_i \cdot \vec{m}_j) - 3(\vec{m}_i \cdot \hat{r})(\vec{m}_j \cdot \hat{r})\right] \quad \hat{r} = \frac{\vec{r}}{r}$$

$$\vec{m}_i = \vec{m}_j$$

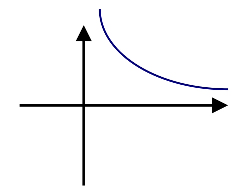
$$U_{ij}^d = \frac{\mu_0 m^2}{4\pi r^3}(1 - 3\cos^2\vartheta) \sim -P_2(\cos\vartheta)$$



$\vartheta < \vartheta_{magic}$

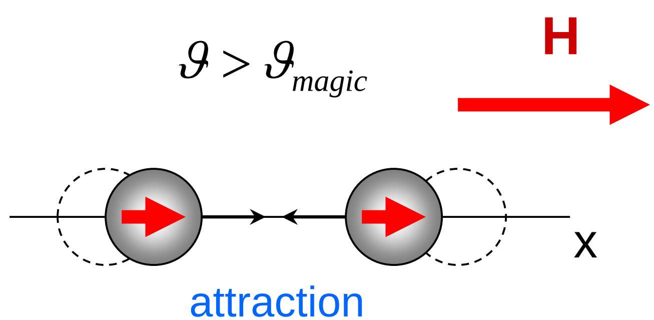


$U_{ij}^d > 0$

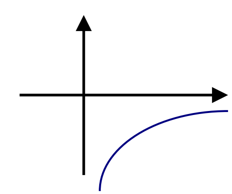


$$\vartheta = \vartheta_{magic} = 54.7^0 \qquad U_{ij}^d = 0$$

$\vartheta > \vartheta_{magic}$



$U_{ij}^d < 0$

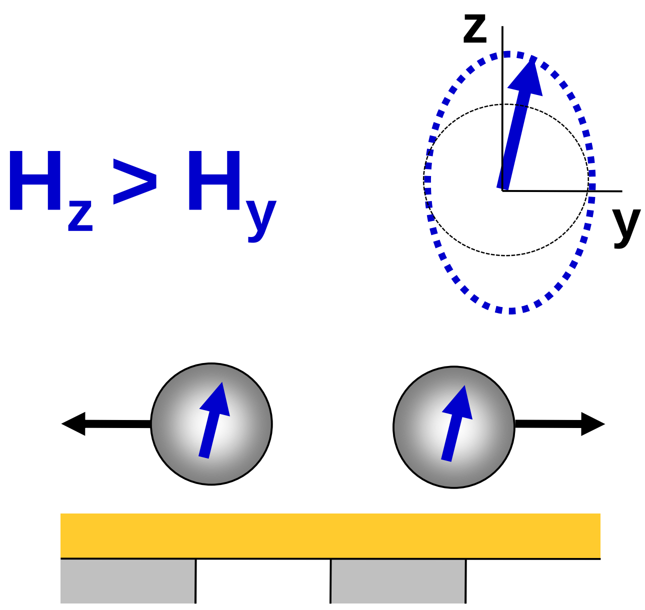


Tuning interactions

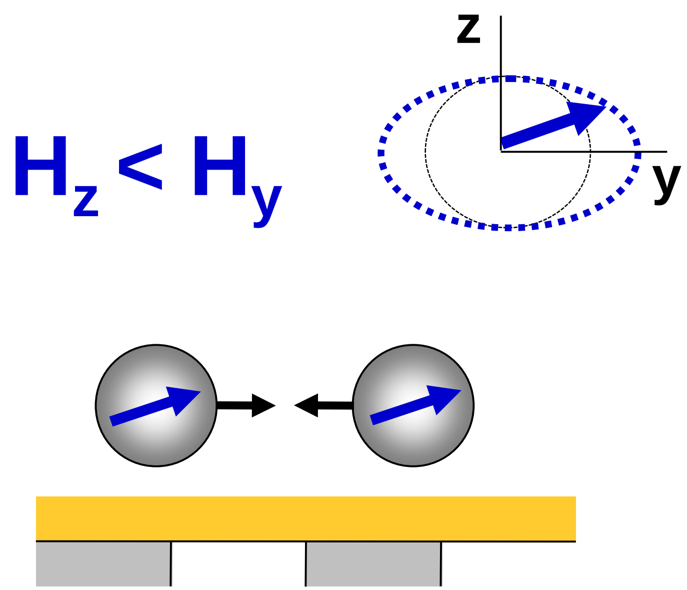
Circularly polarized field → <u>negligible interaction</u>

Elliptically polarized field

Repulsive particles

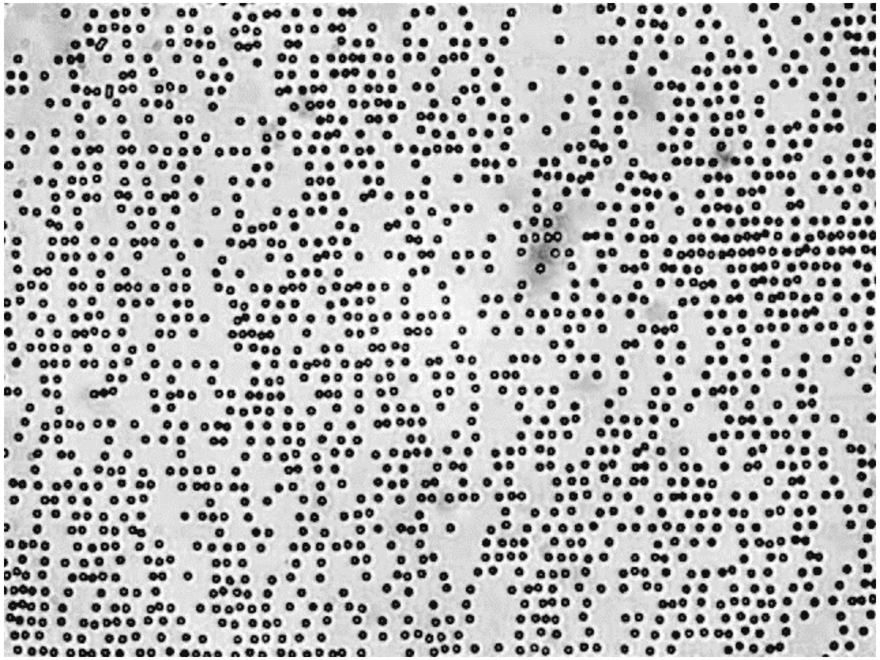
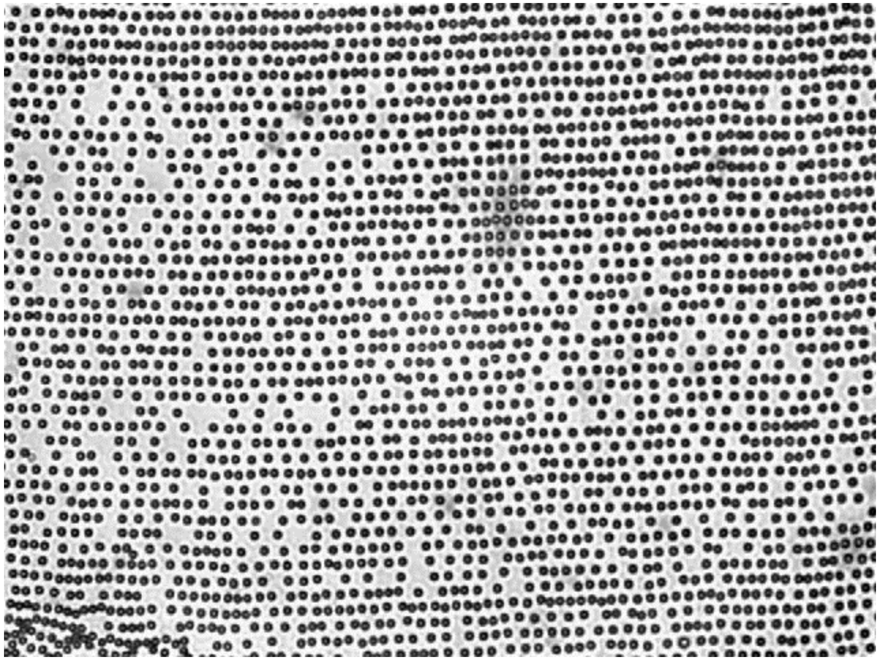


Chains

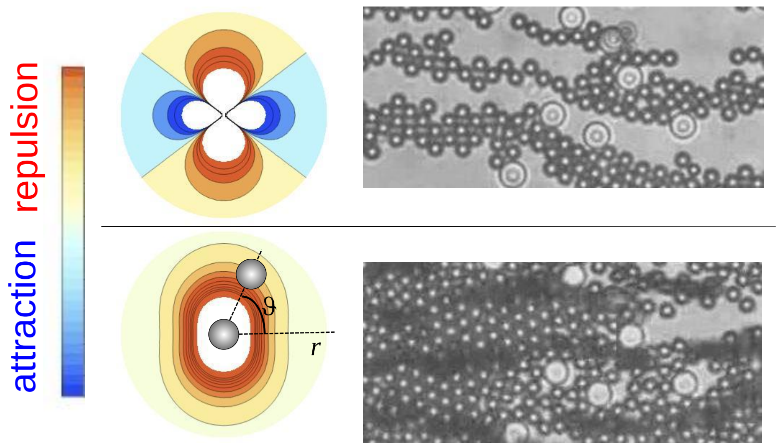


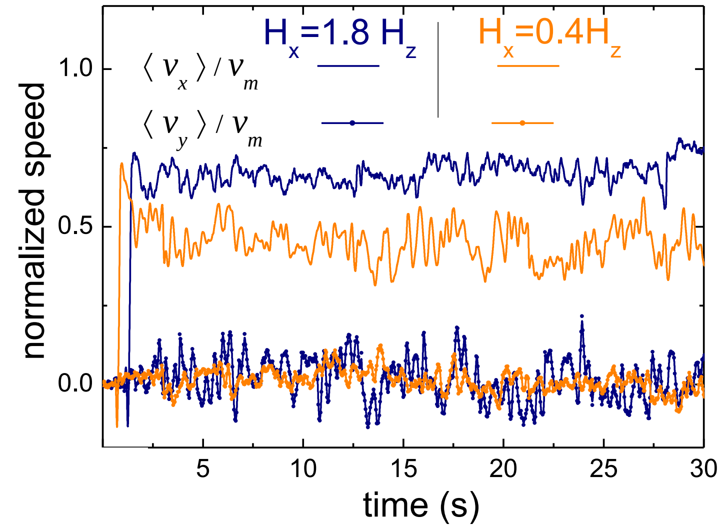
Tuning interactions

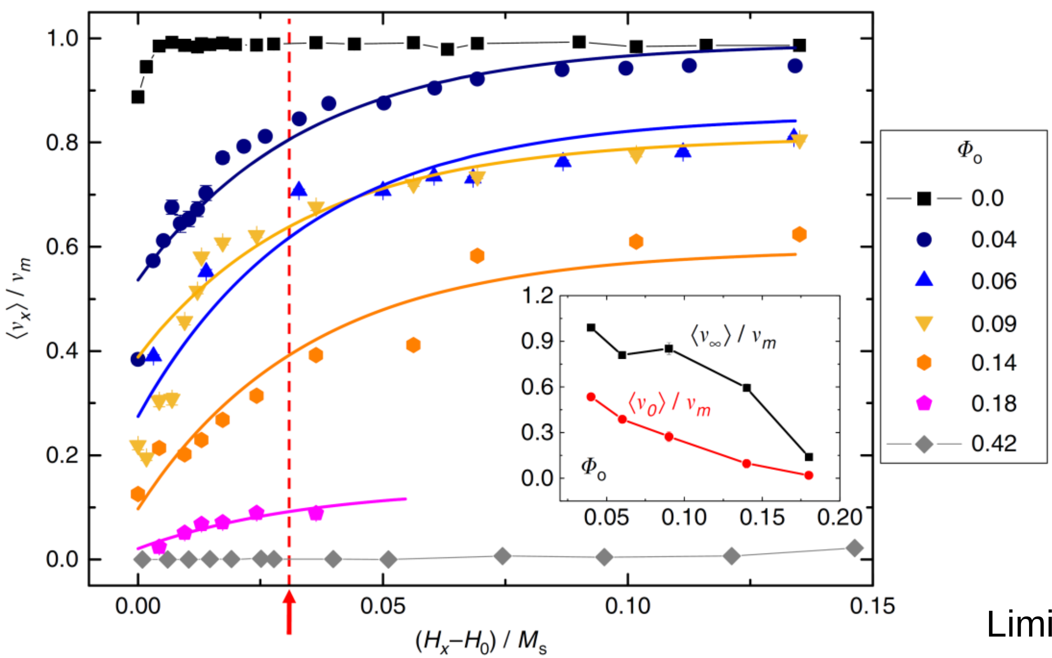
Magnetic dipolar interactions

Repulsive particles	Chains
	

With quenched disorder







Exponential ansatz for the speed

$$\langle v_x \rangle = A\left(1 - e^{-\frac{H_x}{H_c}}\right) + \langle v_0 \rangle$$

$$H_c = H_z/\sqrt{2}$$

$$A = \langle v_0 \rangle - \langle v_\infty \rangle$$

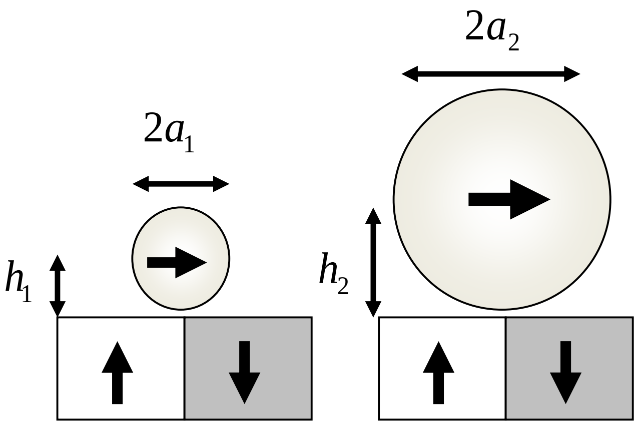
Limit velocities for small and large H_x

Bidirectional transport (with Arthur Straube, FU Berlin)

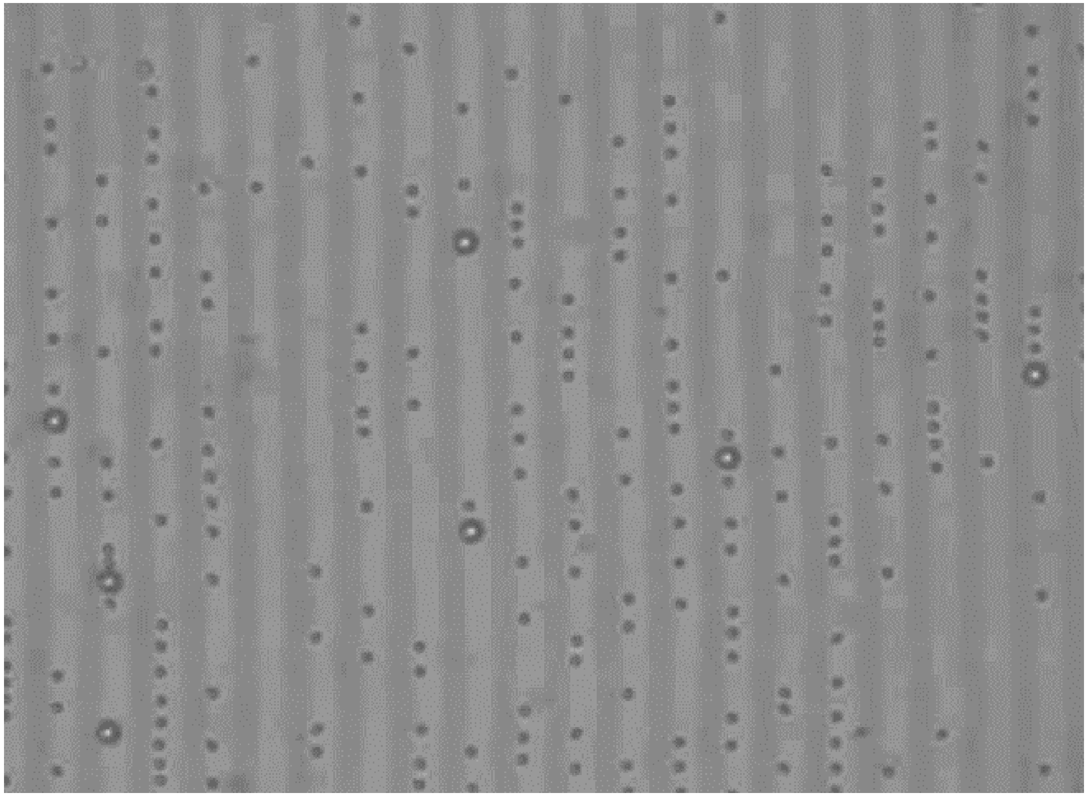
$a_{1,2}$ = particle radius

$F_{p,1,2}$ = pinning force ~ 100 pN.

$h_{1,2}$ = distance from surface

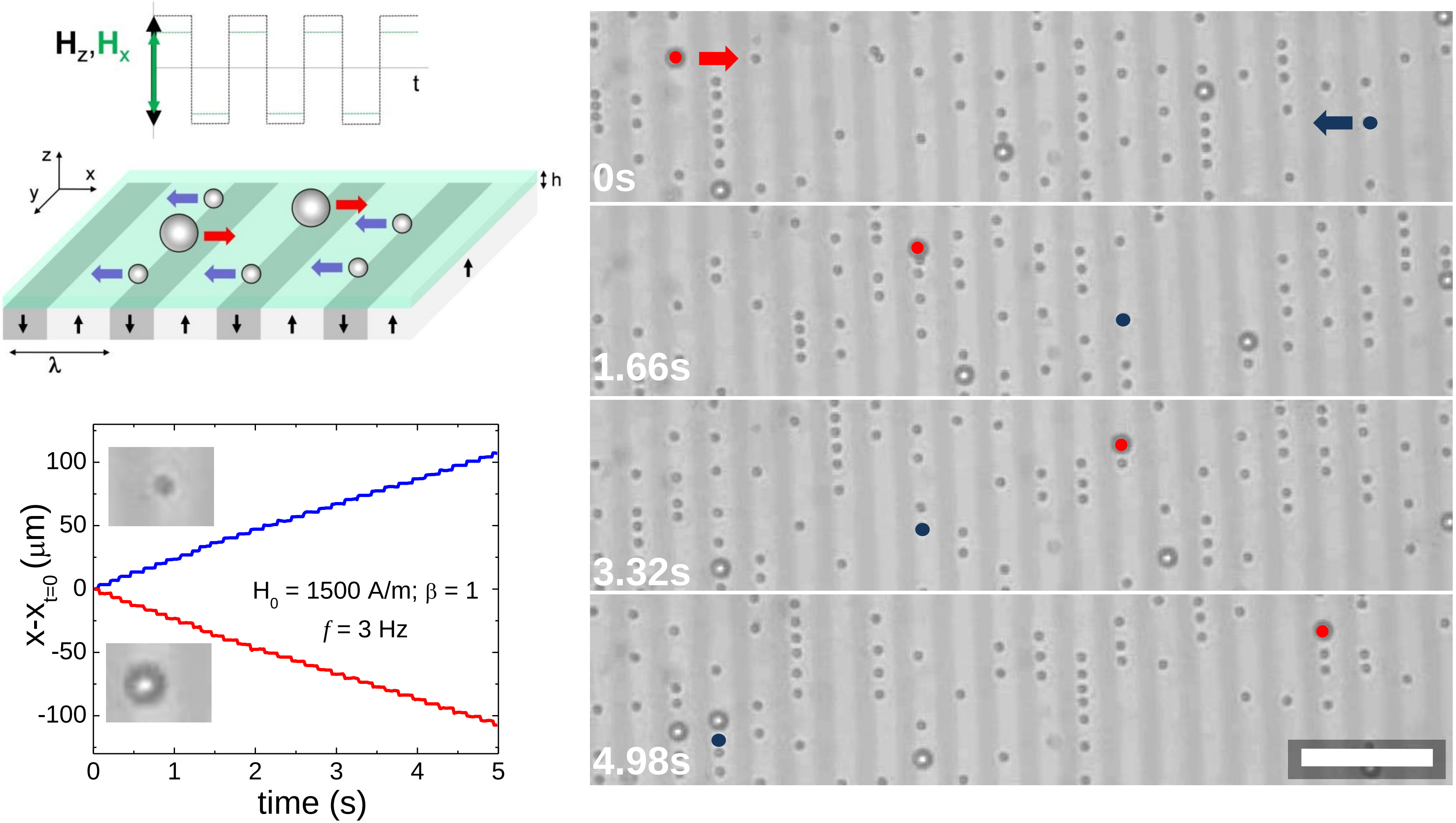


$a_1 < a_2 \Rightarrow h_1 < h_2 \Rightarrow F_{p,1} > F_{p,2}$



Bidirectional transport (with Arthur Straube, FU Berlin)

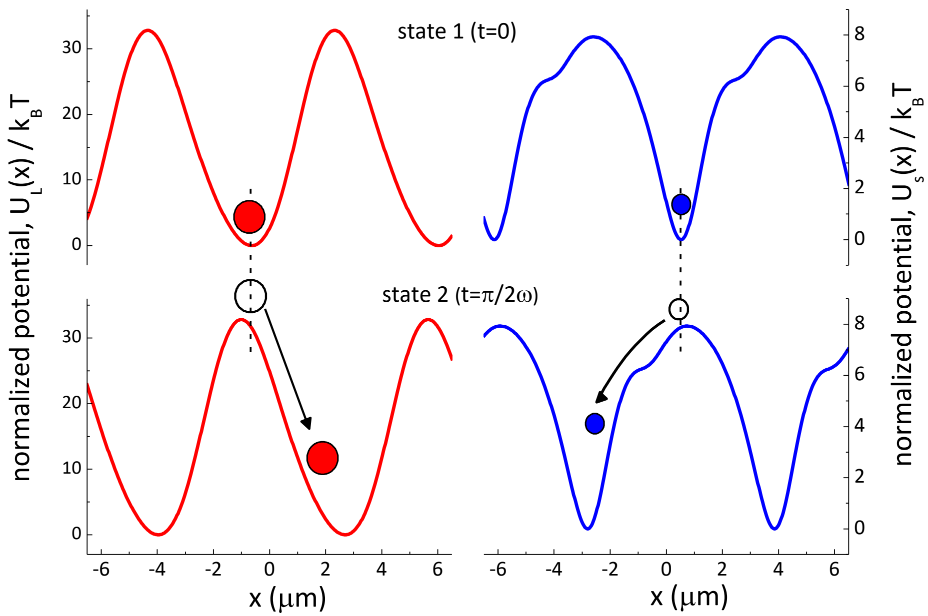
$$\mathbf{H}^{ac} \equiv [H_x \mathrm{sgn}(\cos(2\pi f t)), 0, H_z \mathrm{sgn}(\cos(2\pi f t))]$$

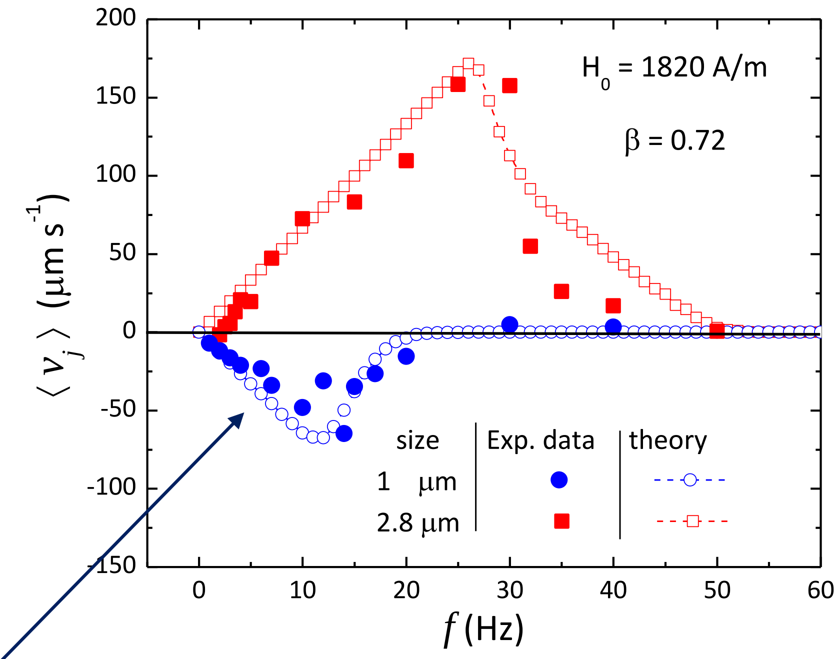


Bidirectional transport and size-selective sorting

$$\mathbf{H}^{ac} \equiv [H_x \mathrm{sgn}(\cos(2\pi f t)), 0, H_z \mathrm{sgn}(\cos(2\pi f t))]$$

Energy landscape explains the transport mechanism



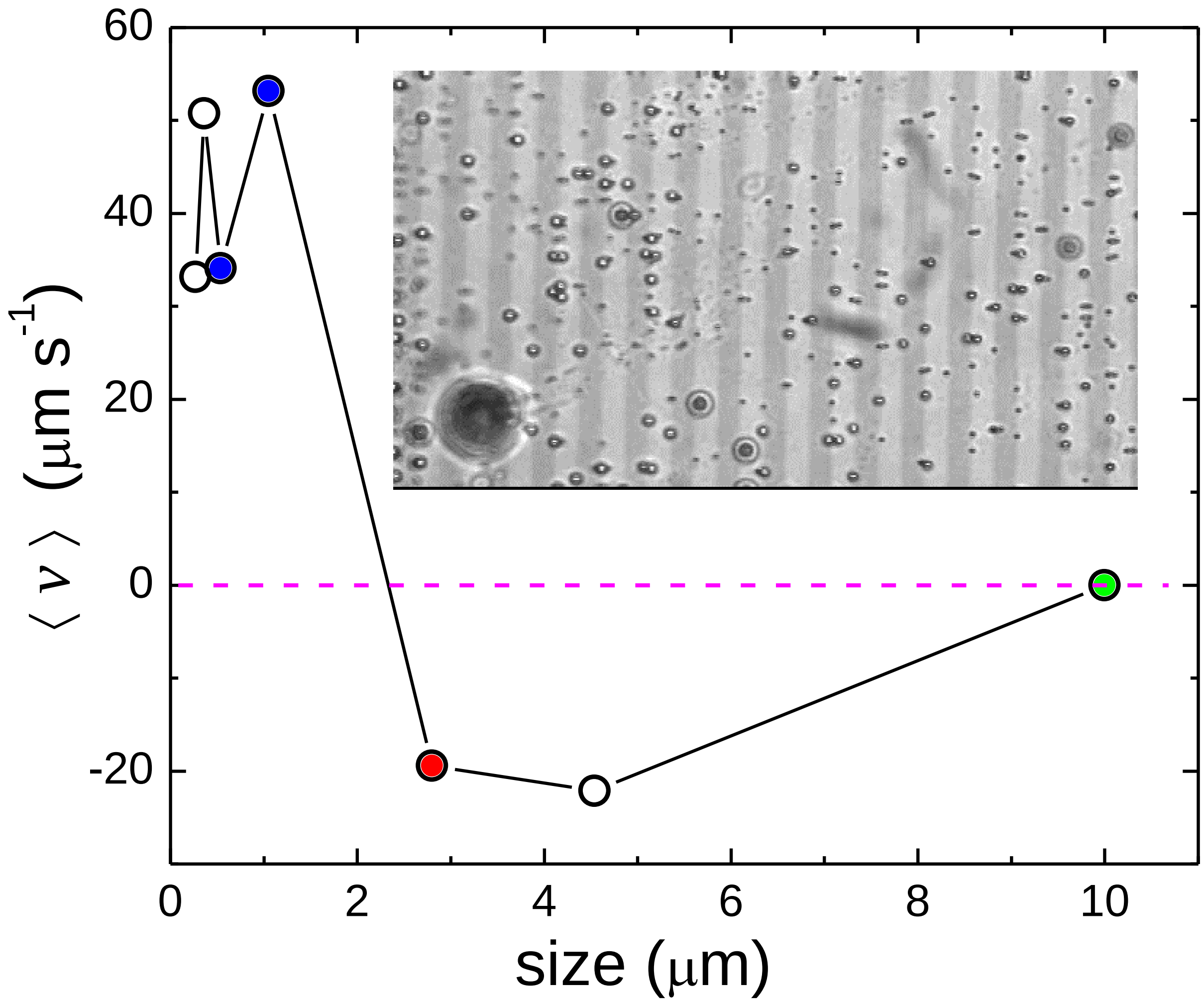


Langevin dynamics

$$\zeta_j \frac{dx_j}{dt} = -\frac{\partial U_j(x_j,t)}{\partial x} + \sqrt{2\zeta_j k_B T}\,\xi_j(t), \quad j = \{\mathrm{s},\mathrm{L}\}$$

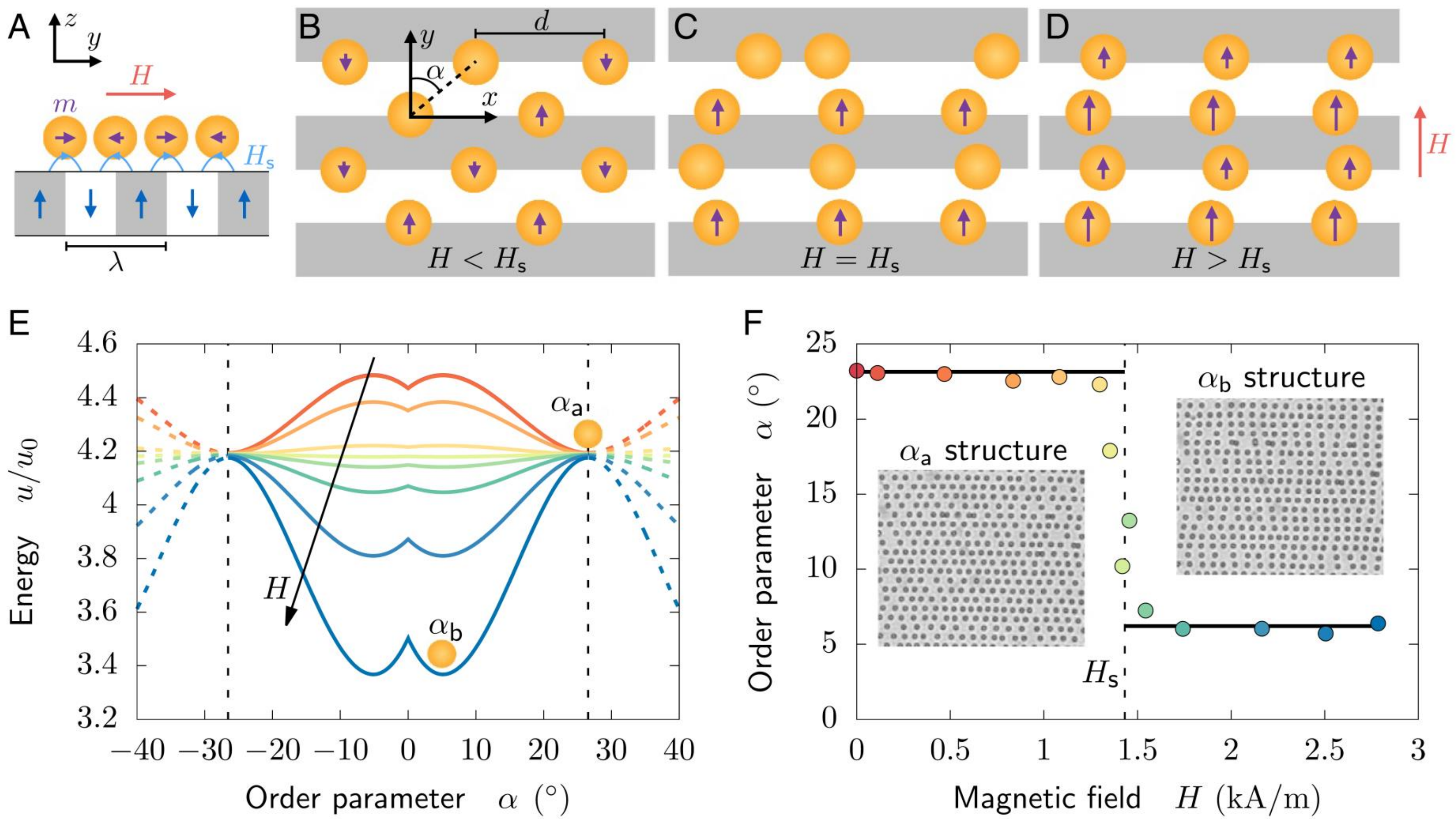
$$U_j(x,t) = \frac{1}{2}(\mu_0 V_j \chi_j)\mathbf{H}^2(x, z_j, t)$$

Bidirectional transport and size-selective sorting



F. Martinez-Pedrero *et al. Phys. Chem. Chem. Phys.*, **18**, 26353 (2016)

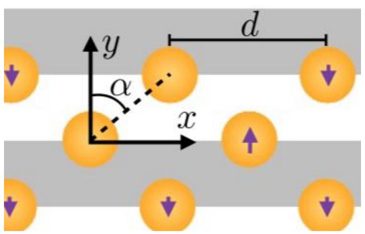
Mixed order phase transition (with R. Alert and J. Casademunt, UB)



Landscape induced pase transition (I order)

R. Alert, J. Casademunt, P. Tierno *PRL* **113**, 198301 (2014)

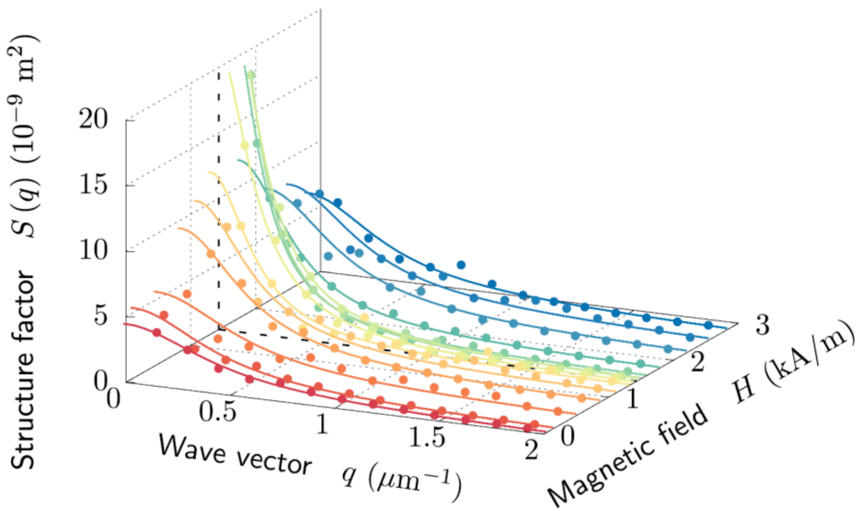
Mixed order phase transition (with R. Alert and J. Casademunt, UB)

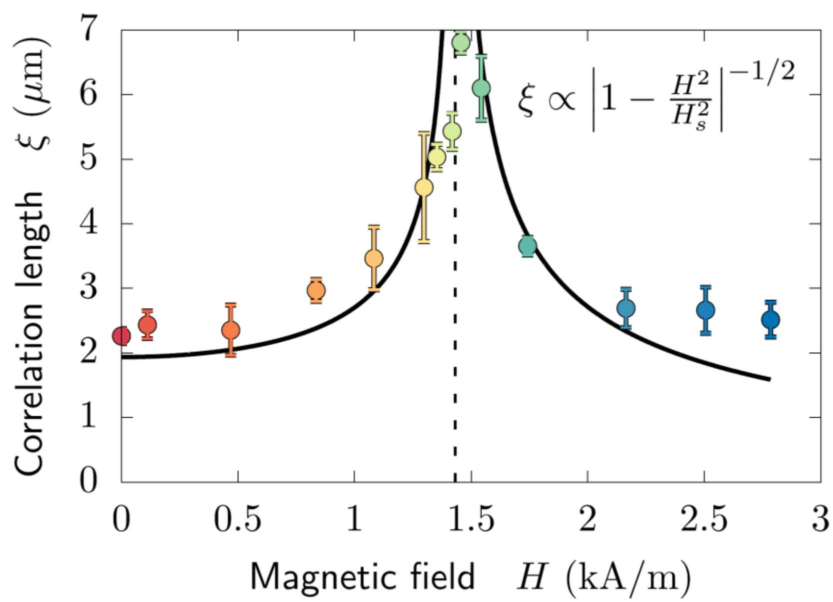


$$C(\vec{r}) = \frac{\left\langle \langle \delta\alpha(\vec{r}\,')\,\delta\alpha(\vec{r}\,'+\vec{r})\rangle_{\vec{r}\,'}\right\rangle_t - \left\langle \langle \delta\alpha(\vec{r}\,')\rangle_{\vec{r}\,'}\right\rangle_t^2}{\left\langle \langle \delta\alpha^2(\vec{r}\,')\rangle_{\vec{r}\,'}\right\rangle_t - \left\langle \langle \delta\alpha(\vec{r}\,')\rangle_{\vec{r}\,'}\right\rangle_t^2}$$

$$S(q) = \int_S C(\vec{r})\, e^{-i\vec{q}\cdot\vec{r}}\, d^2\vec{r}.$$

correlation length diverges at the transition point **(II order!)**





$$S(q) = \langle |\delta\tilde{\alpha}_{\vec{q}}|^2 \rangle = \frac{R^{-2}}{q^2 + \xi^{-2}}$$

$$\xi = \left[\frac{\kappa}{\bar{u}\,(1 - H^2/H_s^2)\,\partial_\alpha^2 f|_{\alpha^*}}\right]^{1/2}.$$

R. Alert, P. Tierno, J. Casademunt *PNAS* **113**, 198301 (2014)

Conclusions

Potential future directions (…ideas are welcome!)

- Dynamics on complex magnetic patterns (bubbles, disorder etc…)
- Transport of biological cargos
- Directional locking effects

Acknowledgements

Scientific discussions:

- Thomas Fischer (Uni. Bayreuth, Germany)
- Tom H. Johansen (Uni. Oslo, Norway)
- Charles Reichhardt (Los alamos Nat. Lab., USA)

Funding agencies

erc

IDEAS

GOBIERNO DE ESPAÑA

MINISTERIO DE EDUCACIÓN Y CIENCIA

AGAUR

Agència de Gestió d'Ajuts Universitaris i de Recerca